ELECTRICAL CIRCUIT ANALYSIS AND SYNTHESIS

Lecture Notes (MR20)

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ELECTRICAL CIRCUIT ANALYSIS AND SYNTHESIS

Course Objectives: This course deals about the network theorems and three phase circuits. It also emphasis on network parameters, synthesis and transient analysis of electrical network. It is the foundation for all courses of the Electrical and Electronics Engineering discipline.

MODULE I:Network Theorems and Magnetic Circuits

Superposition, Reciprocity, Thevenin‘s, Norton‘s, Maximum Power Transfer, Millman‘s and Compensation and Tellegen‘s theorems - Statement of theorems and numerical problems in DC and AC Networks.

Magnetic Circuits – Faraday‘s laws of electromagnetic induction – concept of self and mutual inductance – dot convention – coefficient of coupling – composite magnetic circuit - Analysis of series and parallel magnetic circuits. Hysteresis and Eddy currents.

MODULEII: Resonance and Three Phase Circuits:

Resonance – Series & parallel circuits, concept of bandwidth and Q factor.

Locus diagrams: Series R-L, R-C, R-L-C Circuits.

Three Phase Circuits: Introduction to three phase circuits – types of connection - Star and delta– Relation between line and phase voltages and currents in balanced systems

– Analysis of balanced and Unbalanced three phase circuits.

MODULE III:Two Port Network Parameters

Open circuit impedance (Z) network parameters, Short circuit admittance(Y) network parameters –Transmission (ABCD) Inverse Transmission ($A^1B^1C^1D^1$) and Hybrid parameters – Relationship between two port network parameters – Reciprocity and Symmetry concepts of two port network parameters.

MODULEIV: Transient Analysis (Both AC & DC Networks

Introduction - Initial conditions of all elements-Transient response of Series R-L, R- C and R-L-C circuits (Independent Sources Only) – Solution using Laplace transform approach.

MODULEV: Network Synthesis

Hurwitz Polynomials, Positive Real Functions, Frequency Response of Reactive One-Port network, Synthesis of Reactive One Port by Fosters Method, Synthesis of Reactive One Port By Cauer Method, Synthesis of RL, RC and LC One Port

Networks by Foster and Cauer Methods.

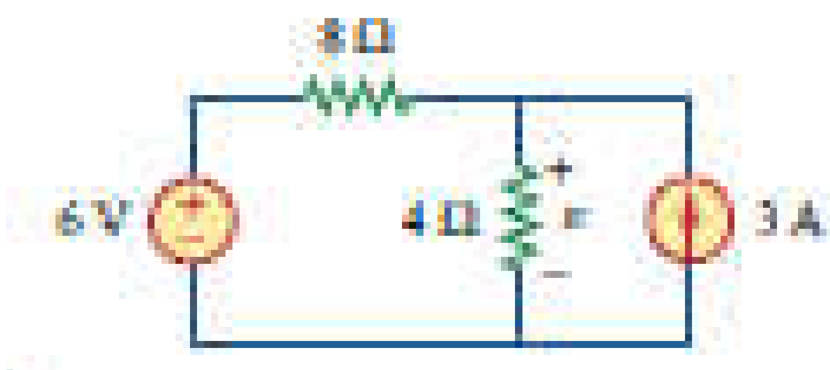
MODULE I
Network Theorems and Magnetic Circuits

Network Theorems Introduction
Network theorems are also can be termed as network reduction techniques. Each and every theorem got its importance of solving network. Let us see some important theorems with DC and AC excitation with detailed procedures.
Electric circuit theorems are always beneficial to help find voltage and currents in multi loop circuits. These theorems use fundamental rules or formulas and basic equations of mathematics to analyze basic components of electrical or electronics parameters such as voltages, currents, resistance, and so on. These fundamental theorems include the basic theorems like Superposition theorem, Tellegen's theorem, Norton's theorem, Maximum power transfer theorem and Thevenin's theorems.
Other group of network theorems which are mostly used in the circuit analysis process includes Compensation theorem, Substitution theorem, Reciprocity theorem, Millman's theorem and Miller's theorem.

SUPERPOSITION THEOREM:

Statement: In an any linear, bi-lateral network consisting number of sources, response in any element (resistor) is given as sum of the individual responses due to individual sources, while other sources are non-operative"

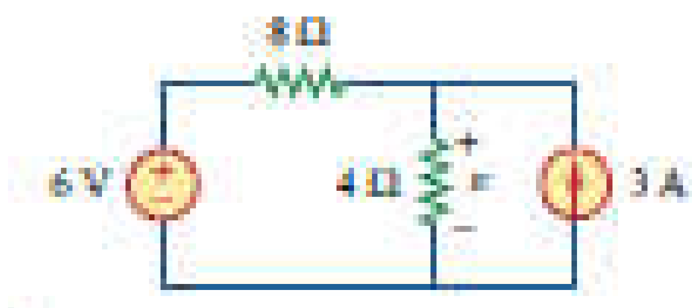
Eg: Let V = 6v, I = 3A, R1 = 8 ohms and R2 = 4 ohms Let us find current through 4 ohms using V source, while I is zero. Then equivalent circuit is



Let i1 is the current through 4 ohms, i1 = V / (R1+R2)

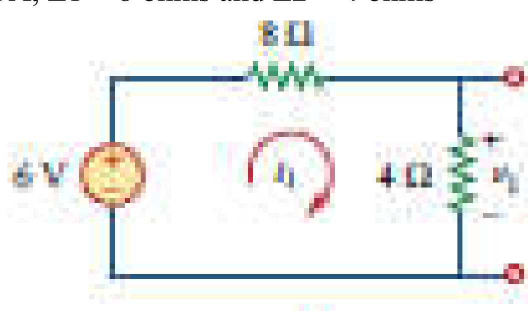
Let us find current through 4 ohms using I source, while V is zero. Then equivalent circuit is

Let i2 is the current through 4 ohms, i2 = I. R1 / (R1+R2)



Hence total current through 4 ohms is = I1+I1 (as both currents are in same direction or otherwise I1-I2)

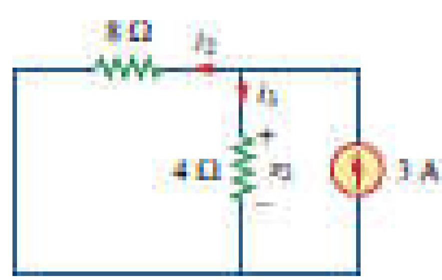
Let V = 6v, I = 3A, Z1 = 8 ohms and Z2 = 4 ohms



Let us find current through 4 ohms using V source, while I is zero. then equivalent circuit is

Let i1 is the current through 4 ohms, i1 = V / (Z1+Z2)

Let us find current through 4 ohms using I source, while V is zero.then equivalent circuit is

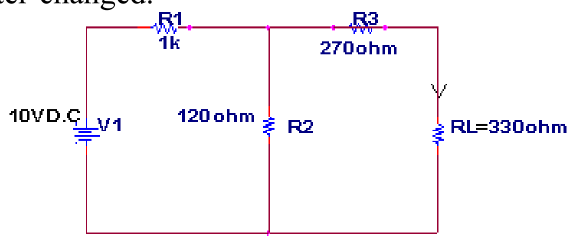


Let i2 is the current through 4 ohms, i2 = I. Z1 / (Z1+Z2)

Hence total current through 4 ohms is = I1+I1(as both currents are in same direction or otherwiseI1-I2).

RECIPROCITY THEOREM:

Statement: In any linear bi-lateral network ratio of voltage in one mesh to current in other mesh is same even if their positions are inter-changed.



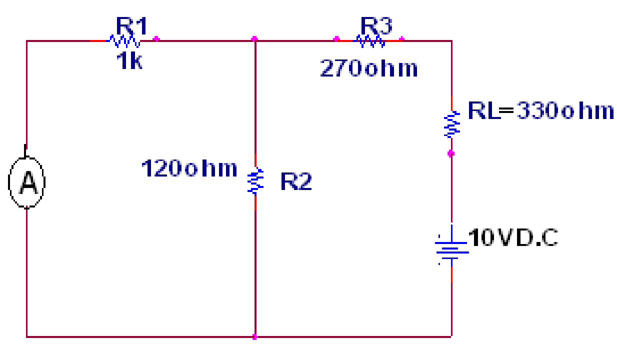
Eg: Find the total resistance of the circuit, Rt = R1+ [R2(R3+Rl)] / R2+R3+RL. Hence

source current, I = V1 / Rt.

Current through RL is I1 = I. R2 / (R2+R3+RL) Take the

ratio of, V1 / I1

Draw the circuit by inter changing position of V1 and I1



Eg: Find the total resistance of the circuit, Rt = (R3+RL) + [R2(Rl)] / R2+R1. Hence

source current, I = V1 / Rt.

Current through RL is I1 = I. R2 / (R2+R1) Take the

ratio of , V1 / I1 ---2

If ratio 1 = ratio 2, then circuit is said to be satisfy reciprocity.

THEVENIN'S THEOREM:

Statement: An complex network consisting of number voltage and current sources cand be replaced by simple series circuit consisting of equivalent voltage source in series with equivalent resistance, where equivalen voltage is called as open circuit voltage and equivalent resistance is called as thevenin's resistance calculated across open circuit terminals while all energy sources are non-operative

Eg: Here we need to find current through RL using thevenin's theorem. Open circuit the AB terminals to find the Thevenin's voltage.Thevenin'svoltage , Vth = E1. R3/ (R1+R3)from figure.1 Thevenin's resistance, Rth = (R1.R3)/ (R1+R3) + R2 2 from figure 2.

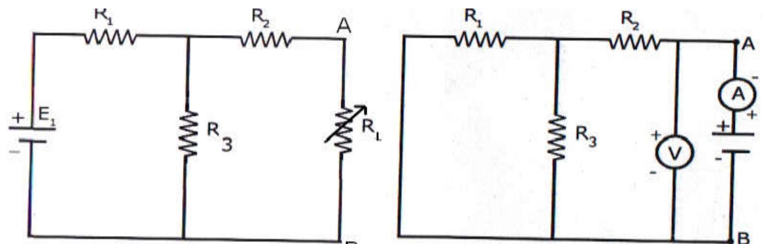


Fig 1 Fig 2

Now draw the thevenin's equivalent circuit as shown in figure 3 with calculated values.

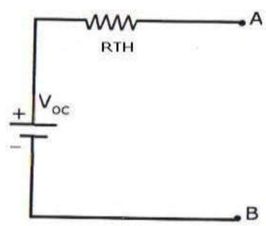


Fig3

NORTON'S THEOREM:

Statement: A complex network consisting of number voltage and current sources cand be replaced by simple parallel circuit consisting of equivalent current source in parallel with equivalent resistance, where equivalent current source is called as short circuit current and equivalent resistance is called as Norton's resistance calculated across open circuit terminals while all energy sources are non-operative

Example:

Here we need to find current through RL using Norton's theorem. Short circuit the AB terminals to find the Norton are current.

Total resistance of circuit is, Rt = (R2.R3) / (R2+R3) + R1 Source current, I = E / Rt

Norton's current, IN = I. R3 / (R2+R3) ----1 from figure .1

Norton's resistance, RN = (R1.R3)/ (R1+R3)+R2 2 from figure2.

Now draw the Norton's equivalent circuit as shown in figure 3 with calculated values.

MAXIMUM POWER TRANSFER THEOREM:

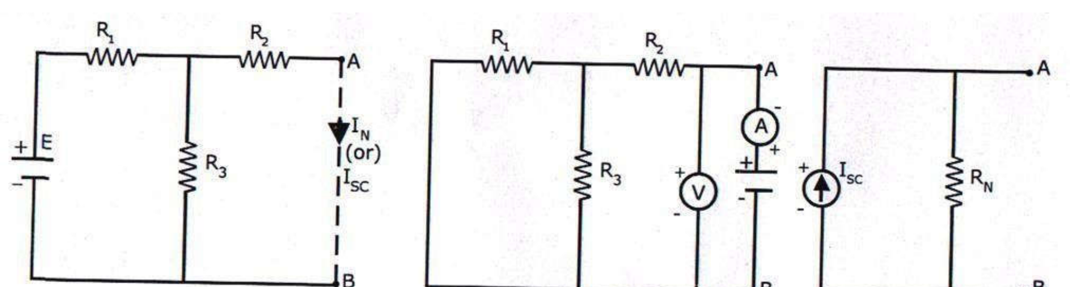
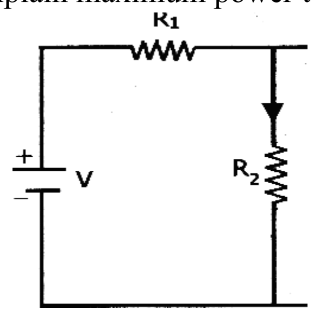


Fig 1 Fig 2 Fig 3

Statement: In linear bi-lateral network maximum power can be transferred from source to load if load resistance is equal to source or thevenin‘s or internal resistances

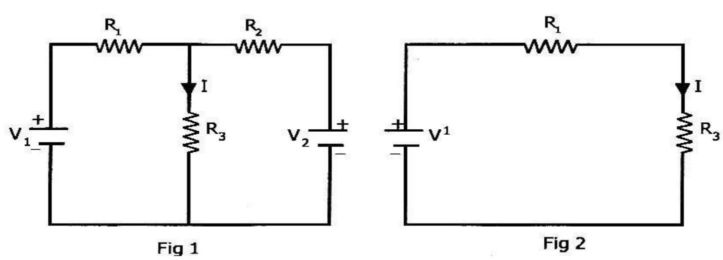
Eg: For the below circuit explain maximum power transfer theorem.



Let I be the source current, I = V / (R1+R2) Power absorbed by load resistor is,
PL = I2 .R2
= [V / (R1+R2)]^2 .R2.
To say that load resistor absorbed maximum power ,dPL / dR2 = 0.
When we solve above condition we get, R2 = R1.
Hence maximum power absorbed by load resistor is, PLmax = V2 / 4R2.

MILLIMAN’S THEOREM:

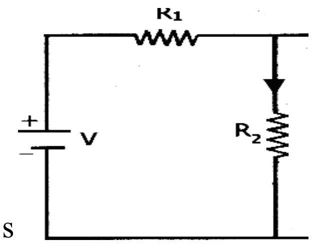
Statement: An complex network consisting of number of parallel branches , where each parallel branch consists of voltage source with series resistance, can be replaced with equivalent circuit consisting of one voltage source in series with equivalent resistance‖



Where equivalent voltage source value is,
V=(V1G1+V2G2+ ------- +VnGn)
G1+G2+.......................Gn
Equivalent resistance is ,R‘=1 /(G1+G2+Gn)

COMPENSATION THEOREM:

Statement: States that any element in the network can be replaced with Voltage source whose value is product of current through that element and its value‖ It is useful in finding change in current when sudden change in resistance value.



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For the above circuit source current is given as, I = V / (R1+R2) Element R2 can be replaced with voltage source of, V = I.R2
Let us assume there is change in R2 by ΔR, now source current is I'= V / (R1+R2+ ΔR) Hence actual change in current from original circuit to present circuit is = I – I'.
This can be find using compensation theorem as, making voltage source non-operative and replacing ΔR with voltage source of I'. ΔR.
Then change in current is given as = I'. ΔR / (R1+R2)

TELLEGENS THEOREM

Dutch Electrical Engineer Bernard D.H. Tellegen has introduced this theorem in the year of 1952. This is a very useful theorem in network analysis. According to Tellegen theorem, the summation of instantaneous powers for the n number of branches in an electrical network is zero. Are you confused? Let's explain. Suppose n number of branches in an electrical network have i1, i2, i3, in respective instantaneous currents through them. These currents satisfy Kirchhoff's Current Law. Again, suppose these branches have instantaneous voltages across them are v1,v2,v3, vn respectively. Ifthese

$$\sum_{k=1}^{n} v_k \cdot i_k = 0$$

Voltages across these elements satisfy Kirchhoff Voltage Law then, v_k is the instantaneous voltage across the k^{th} branch and i_k is the instantaneous current flowing through this branch.

Tellegen theorem is applicable mainly in general class of lumped network s that consist of linear, non-linear, active, passive,

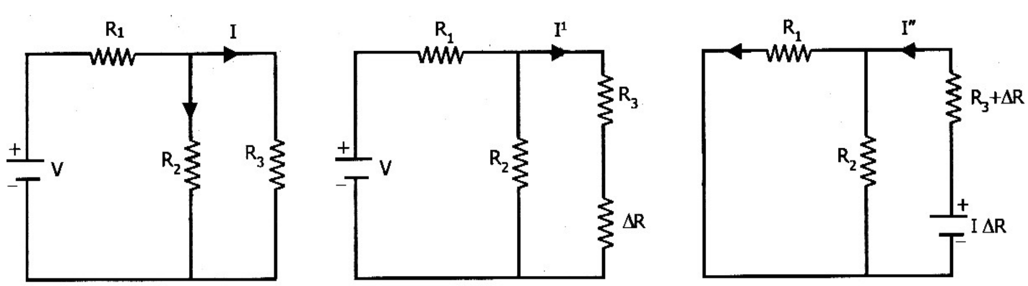
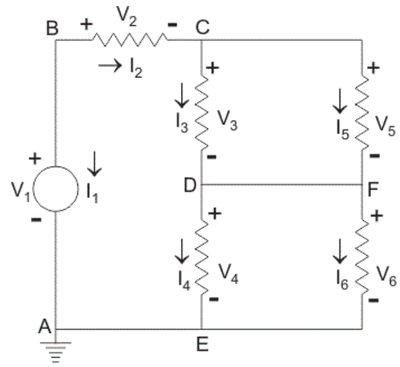


Fig 1 Fig 2 Fig 3

time variant and time variant elements. This theorem can easily be explained by the following example.



In the network shown, arbitrary reference directions have been selected for all of the branch currents, and the corresponding branch voltages have been indicated, with positive reference direction at the tail of the current arrow.

For this network, we will assume a set of branch voltages satisfy the Kirchhoff voltage law and a set of branch current satisfy Kirchhoff current law at each node. We will then show that these arbitrary assumed

$$\sum_{k=1}^{n} v_k \cdot i_k = 0$$

Voltages and currents satisfy the equation. And it is the condition of Tellegen theorem. In the network shown in the figure, let v1, v2 and v3 be 7, 2 and 3 volts respectively. Applying Kirchhoff voltage law around loop ABCDEA. We see that v4 = 2 volt is required. Around loop CDFC, v5 is required to be 3 volt and around loop DFED, v6 is required to be 2. We next apply Kirchhoff current law successively to nodes B, C and D. At node B let i_i= 5 A, then it is required that i2 = − 5 A. At node C let i3 = 3 A and then i5 is required to be − 8. At node D assume i4 to be4then

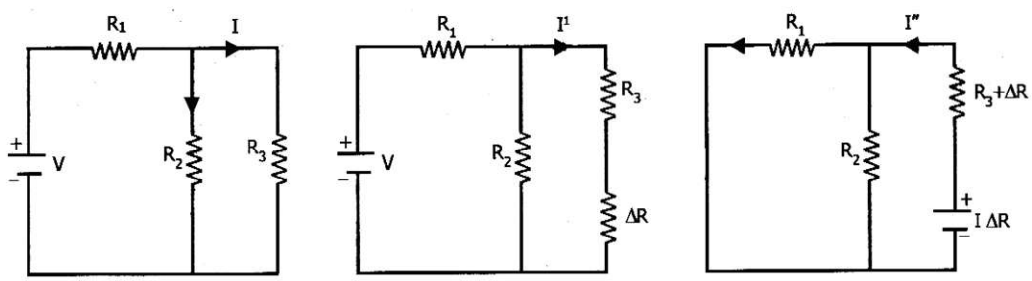


Fig 1 Fig 2 Fig 3

$\overline{k=1}$

i6is required to be − 9. Carrying out the operation of equation, we get,

$$7 \times 5 + 2 \times (-5) + 3 \times 3 + 2 \times 4 + 3 \times (-8) + 2 \times (-9) = 0$$

Hence Tellegen theorem is verified.

Magnetic Circuits

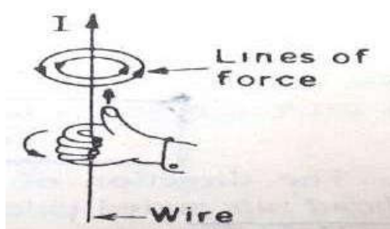
Magnetic fields are the fundamental medium through which energy is converted from one form to another in motors, generators and transformers. Four basic principles describe how magnetic fields are used in these devices.

1. A current-carrying conductor produces a magnetic field in the area around it. Explained in Detail by Fleming's Right hand rule and Amperes Law.
2. A time varying magnetic flux induces a voltage in a coil of wire if it passes through that coil.(basis of Transformer action). Explained in detail by the Faradays laws of Electromagnetic Induction.
3. A current carrying conductor in the presence of a magnetic field has a force induced in it (Basis of Motor action)
4. A moving wire in the presence of a magnetic field has a voltage induced in it (Basis of Generator action)

Two basic laws governing the production of a magnetic field by a current carrying conductor :

The direction of the magnetic field produced by a current carrying conductor is given by the Flemings Right hand rule and its' amplitude is given by the *Ampere's Law*.

Flemings right hand rule:
Hold the conductor carrying the current in your right hand such that the Thumb points along the wire in the direction of the flow of current, then the fingers will encircle the wire along the lines of



the Magnetic force

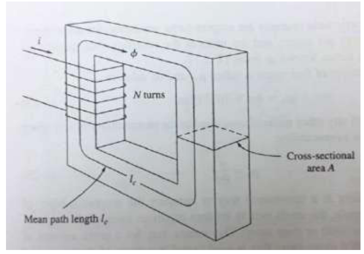
Ampere's Law :
The line integral of the magnetic field intensity H around a closed magnetic path is equal to the total current enclosed by the path.
This is the basic law which gives the relationship between the Magnetic field Intensity H and the current I and is mathematically expressed as

$$\oint H \cdot dl = I_{net}$$

Where H is the magnetic field intensity produced by the current I_{net} and *dl* is a differential element of length along the path of integration. H is measured in Ampere-turns per meter.

Important parameters and their relation in magnetic circuits :

- Consider a current carrying conductor wrapped around a ferromagnetic core as shown in the figure below.



- Applying Ampere's law, the total amount of magnetic field induced will be proportional to the amount of current flowing through the conductor wound with N turns around the ferromagnetic material as shown. Since the core is made of ferromagnetic material, it is assumed that a majority of the magnetic field will be confined to the core.
- The path of integration in this case as per the Ampere's law is the mean path length of the core, l_C. The current passing within the path of integration I_{net} is then Ni, since the coil of wire cuts the path of integration N times while carrying the current i. Hence Ampere's Law becomes: $Hl_C = Ni$
  Therefore
  $$H = Ni/l_C$$
- In this sense, H (Ampere turns per meter) is known as the effort required to induce a magnetic field. The strength of the magnetic field flux produced in the core also depends on the material of the core. Thus: $B = \mu H$
  Where,
  B = magnetic flux density [webers per square meter, or Tesla (T)]
  μ= magnetic permeability of material (Henrys per meter)
  H = magnetic field intensity (ampere-turns per meter)
- The constant μ may be further expanded to include *relative permeability* which can be defined as below:
  $\mu_r = \mu/\mu_o$
  where ,μ_o = permeability of free space (equal to that of air)
- Hence the permeability value is a combination of the relative permeability and the permeability of free space. The value of relative permeability is dependent upon the type of material used. The higher the amount permeability, the higher the amount of flux induced in the core. Relative permeability is a convenient way to compare the magnetic ability of materials.
- Also, because the permeability of iron is so much higher than that of air, the majority of the flux in an iron core remains inside the core instead of travelling through the surrounding air, which has lower permeability. The small leakage flux that does leave the iron core is important in determining the flux linkages between coils and the self-inductances of coils in transformers and motors.
- In a core such as shown in the figure above
  $$B = \mu H = \mu\ Ni/l_C$$
  Now, to measure the total flux flowing in the ferromagnetic core, consideration has to be made in terms of its cross sectional area (CSA). Therefore:

Φ = ∫ □. □□where: A = cross sectional area throughout the core.

Assuming that the flux density in the ferromagnetic core is constant through out hence the equation simplifies to:

$$\Phi = B.A$$

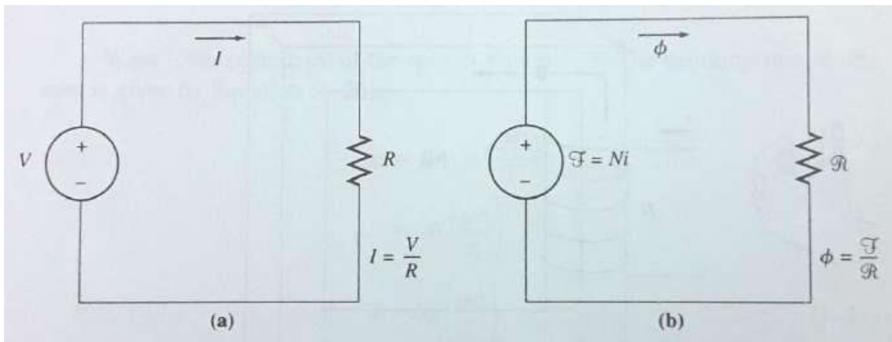
Taking the previous expression for B

we *get $\Phi = \mu NiA/l$*

Electrical analogy of magnetic circuits:

The flow of magnetic flux induced in the ferromagnetic core is analogous to the flow of electric current in an electrical circuit hence the name magnetic circuit.

The analogy is as follows:



(a) Electric Circuit (b) Electrical Analogy of Magnetic Circuit

- Referring to the magnetic circuit analogy, F is denoted as magneto motive force (mmf) which is similar to Electromotive force in an electrical circuit (emf). Therefore, we can say that F is the force which pushes magnetic flux around a ferromagnetic core with a value of Ni (refer to ampere's law). Hence F is measured in ampere turns. Hence the magnetic circuit equivalent equation is a shown:

  F = Ø.R (similar to V=IR)

  We already have the relation Φ = μ NiA/l and using this we get R = F / Φ = Ni/ Φ

  R = Ni /(μ NiA/l) = l/ μ A
- The polarity of the mmf will determine the direction of flux. To easily determine the direction of flux, the *'right hand curl'* rule is applied:

  When the direction of the curled fingers indicates the direction of current flow the resulting thumb direction will show the magnetic flux flow.
- The element of R in the magnetic circuit analogy is similar in concept to the electrical resistance. It is basically the measure of material resistance to the flow of magnetic flux. Reluctance in this analogy obeys the rule of electrical resistance (Series and Parallel Rules). Reluctance is measured in Ampere-turns per Weber.
- The inverse of electrical resistance is conductance which is a measure of conductivity of a material. Similarly the inverse of reluctance is known as permeance P which represents the degree to which the material permits the flow of magnetic flux.
- By using the magnetic circuit approach, calculations related to the magnetic field in a ferromagnetic material are simplified but with a little in accuracy

Equivalent Reluctance of a series Magnetic circuit: Req series=R1 + R2 + R3 +….

Equivalent Reluctance of a Parallel Magnetic circuit: $1/R_{eq\ parallel}=1/R_1 + 1/R_2 + 1/R_3 +\ldots$

Electromagnetic Induction and Faraday's law Induced Voltage from a Time-Changing Magnetic Field:

Faraday's Law:

Whenever a varying magnetic flux passes through a turn of a coil of wire, voltage will be induced in the turn of the wire that is directly proportional to the rate of change of the flux linkage with the turn of the coil of wire.

$$E_{ind} \propto -dØ/dt$$

$$E_{ind}=-\square .\ dØ/dt$$

The negative sign in the equation above is in accordance to Lenz' Law which states:

The direction of the induced voltage in the turn of the coil is such that if the coil is short circuited, it would produce a current that would cause a flux which opposes the original change of flux.

And k is the constant of proportionality whose value depends on the system of units chosen. In the SI system of units k=1 and the above equation becomes:

$$E_{\ ind}= - dØ/dt$$

Normally a coil is used with several turns and if there are N number of turns in the coil with the same amount of flux flowing through it then:

$E_{\ ind} = - \square dØ/dt$

Change in the flux linkage NØ of a coil can be obtained in two ways:

1. Coil remains stationary and flux changes with time (Due to AC current like in Transformers and this is called Statically induced e.m.f
2. Magnetic flux remains constant and stationary in space, but the coil moves relative to the magnetic field so as to create a change in the flux linkage of the coil (Like in Rotating machines and this is a called Dynamically induced e.m.f.

Self inductance:

From the Faradays laws of Electromagnetic Induction we have seen that an e.m.f will be induced in a conductor when a time varying flux is linked with a conductor and the amplitude of the induced e.m.f is proportional to the rate of change of the varying flux.

If the time varying flux is produced by a coil of N turns then the coil itself links with the time varying flux produced by itself and an emf will be induced in the same coil. This is called self inductance .

The flux Ø produced by a coil of N turns links with its own N turns of the coil and hence the total flux linkage is equal to $NØ = (\mu N^2 A / l)\ I$ *[using the expression $\Phi = \mu NiA/l$* we already developed] Thus we see that the total magnetic flux produced by a coil of N turns and linked with itself is proportional to the current flowing through the coil i.e.

$$NØ \propto \text{or} \quad NØ = L\ \square$$

From the Faradays law of electromagnetic Induction, the self induced e.m.f for this coil of N turns is given by:

$$E_{\ ind}= - \square\ dØ/dt = -L\ dI/dt$$

The constant of proportionality L is called the self Inductance of the coil or simply Inductance and its value is given by $L = (\mu N^2 A / l)$. If the radius of the coil is r then:

$$L = (\mu N^2 \pi r^2 / l)\ i$$

From the above two equations we can see that Self Inductance of a coil can be defined as the flux produced per unit current i.e *Weber /Ampere* (equation1) or the induced emf per unit rate of change of current i.e *Volt-sec/Ampere* (equation 2)

The unit of Inductance is named after Joseph Henry as *Henry* and is given to these two combinations as :

$1H = 1WbA^{-1} = 1VsA^{-1}$

Self Inductance of a coil is defined as one Henry if an induced emf of one volt is generated when the current in the coil changes at the rate of one Ampere per second.

Henry is relatively a very big unit of Inductance and we normally use Inductors of the size of mH(10^{-3} H) or µH (10^{-3}H)

Mutual inductance and Coefficient of coupling:

In the case of Self Inductance an emf is induced in the same coil which produces the varying magnetic field. The same phenomenon of Induction will be extended to a separate second coil if it is located in the vicinity of the varying magnetic field produced by the first coil. Faradays law of electromagnetic Induction is equally applicable to the second coil also. A current flowing in one coil establishes a magnetic flux about that coil and also about a second coil nearby but of course with a lesser intensity. The time-varying flux produced by the first coil and surrounding the second coil produces a voltage across the terminals of the second coil. This voltage is proportional to the time rate of change of the current flowing through the first coil.

Figure (*a)* shows a simple model of two coils $L1$ and $L2$, sufficiently close together that the flux produced by a current $i1(t)$ flowing through $L1$ establishes an open-circuit voltage $v2(t)$ across the terminals of $L2$.*Mutual inductance,M*21, is defined such that

$$v2(t)=M21di1(t)/dt \text{ ------------------ [1]}$$

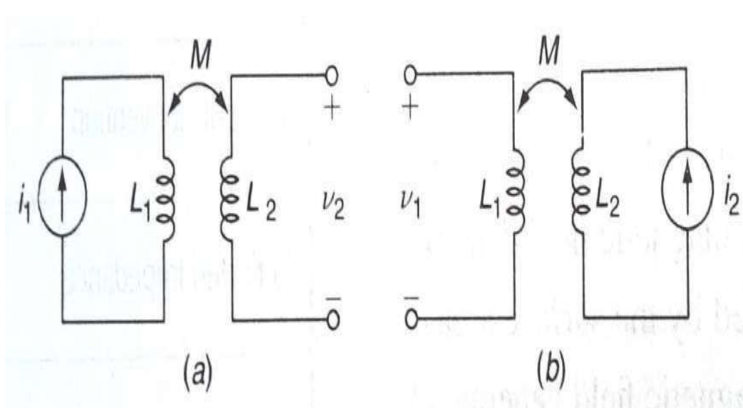


Figure (a) A current $i1$ through $L1$ produces an open-circuit voltage $v2$across $L2$. (b) A current $i2$ through $L2$ produces an open-circuit voltage $v1$ across $L1$.

The order of the subscripts on $M21$ indicates that a voltage response is produced at $L2$ by a current source at $L1$. If the system is reversed, as indicated in fig.(b) then we have

$v1(t)=M12di2(t)/dt$ ----------------[2]

It can be proved that the two mutual inductances $M12$ and $M21$ are equal and thus, M12 = M21=M

The existence of mutual coupling between two coils is indicated by a double-headed arrow, as shown in Fig. (a)and (b)

Mutual inductance is measured in Henrys and, like resistance, inductance, and capacitance, is a positive quantity. The voltage $M\, di/dt$, however, may appear as either a positive or a negative quantity depending on whether the current is increasing or decreasing at a particular instant of time.

Coefficient of coupling k **:** Is given by the relation $M = k\sqrt{L1\ L2}$ and its value lies between 0 and 1. It can assume the maximum value of 1 when the two coils are wound on the same core such that flux produced by one coil completely links with the other coil. This is possible in well designed cores with high permeability. Transformers are designed to achieve a coefficient of coupling of1.Dot Convention:

The polarity of the voltage induced in a coil depends on the sense of winding of the coil. In the case of Mutual inductance it is indicated by use of a method called "*dot convention*". The dot convention makes use of a large dot placed at one end of each of the two coils which are mutually coupled. Sign of the mutual voltage is determined as follows:

A current entering the dotted terminal of one coil produces an open circuit voltage with a positive voltage reference at the dotted terminal of the second coil.

Thus in Fig(a) $i1$ enters the dotted terminal of $L1$, $v2$ is sensed positively at the dotted terminal of $L2$, and $v2 = M\, di1/dt$

It may not be always possible to select voltages or currents throughout a circuit so that the passive sign convention is everywhere satisfied; the same situation arises with mutual coupling. For example, it may be more convenient to represent $v2$ by a positive voltage reference at the un dotted terminal, as shown in Fig (b). Then $v2 = -M\, di1/dt$. Currents also may not always enter the dotted terminal as indicated by Fig (c) and (d). Then we note that:

A current entering the undotted terminal of one coil provides a voltage that is positively sensed at the undotted terminal of the second coil.

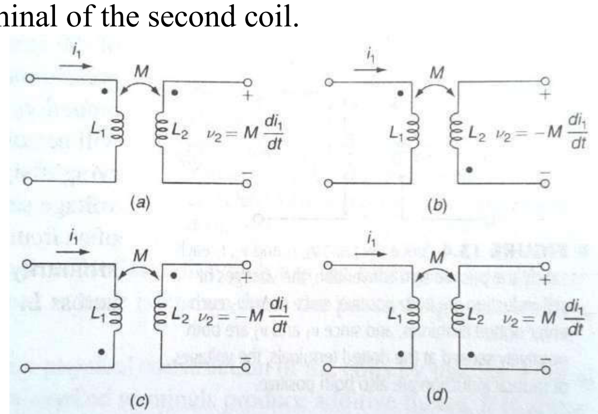


Figure : (a) and (b) Current entering the dotted terminal of one coil produces a voltage that is sensed positively at the dotted terminal of the second coil. (c) and (d) Current entering the undotted terminal of one coil produces a voltage that is sensed positively at the undotted terminal of the second coil.

Important Concepts and formulae:

Resonance and Series RLC circuit:

$\omega r^2 = \omega 1\omega 2 = 1/LC$ ∴

$\omega r = \sqrt{\omega 1\omega 2} = /\sqrt{LC}$ BW =R/2πL

$Q = \omega_r L/R = 1/\omega_r RC$and in terms of R,LandC= $(1/R)(\sqrt{L/C})$

$Q = f_r/BW$ i.e.inversely proportional to theBW

Voltage magnification Magnification = $Q = V_L/V$ or V_C/V

Important points In Series RLC circuit at resonant frequency :

- The impedance of the circuit becomes purely resistive and minimum i.eZ =R
- The current in the circuit becomesmaximum
- The magnitudes of the capacitive Reactance and Inductive Reactance becomeequal
- The voltage across the Capacitor becomes equal to the voltage across the Inductor at resonance and is Q times higher than the voltage across theresistor

Resonance and Parallel RLC circuit:

$\omega r^2 = \omega 1\omega 2 = 1/LC$ ∴ $\omega r = \sqrt{\omega 1\omega 2} = 1/\sqrt{LC}$ same as in series RLCcircuit

BW = 1/2π RC

$Q = R/\omega_r L = \omega_r RC$ and in terms of R, LandC $= R(\sqrt{C/L})$ [Inverse of what we got in Series RLCcircuit]

$Q = f_r/BW$ In Parallel RLC also inversely proportional to the BW

Current Magnification = $Q = I_L/I$ or I_C/I

Important points In Parallel RLC circuit at resonant frequency :

- The impedance of the circuit becomes resistive and maximum i.eZ =R
- The current in the circuit becomesminimum
- The magnitudes of the capacitive Reactance and Inductive Reactance becomeequal
- The current through the Capacitor becomes equal and opposite to the current through the Inductor at resonance and is Q times higher than the current through theresistor

Magnetic circuits :

Ampere'sLaw: $\oint \square . \square\square = I_{net}$ and in the case of a simple closed magnetic path of a ferromagnetic material it simplifies to $Hl=Ni$ or $H = Ni/l$

Magnetic flux density: $B = \mu H$
Magnetic field intensity: $H = Ni/l$
Total magnetic flux intensity: $Ø = BA = \mu HA = \mu \, Ni \, A/l$
Reluctance of the magneticcircuit: $R = mmf/Flux = Ni/Ø = l/\mu A$

Faradays law of electromagnetic Induction:

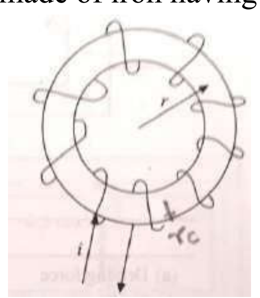
Self induced e.m.f of a coil of N turns is givenby: $e_{ind} = -\square dØ/dt = -L \, dI/dt$ where L is the inductance of the coil of N turns with radius r and given by $L = (\mu N^2 \pi r^2 / l)i$ Equivalent Reluctance of a series Magneti ccircuit: $R_{eq\ series} = R_1 + R_2 + R_3 + \ldots$ Equivalent Reluctance of a Parallel Magnetic circuit: $1/R_{eq\ parallel} = 1/R_1 + 1/R_2 + 1/R_3 + \ldots$ Coefficient of coupling k Is given by the relation:

$$M = k\sqrt{L_1 L_2}$$

Illustrative examples:

Example 1: A toroidal core of radius 6 cms is having 1000 turns on it. The radius of cross section of the core 1cm.Find the current required to establish a total magnetic flux of 0.4mWb.When

(a) The core isnonmagneticThe core is made of iron having a relative permeability of4000



Solution:

This problem can be solved by the direct application of the following formulae we know in magnetic circuits: $B = \Phi/A = \mu H$ and $H = Ni/l$

Where

B = magnetic flux density(Wb/mtr^2) Φ = Total magnetic flux(Wb))

A = Cross sectional area ofthe core(mtr^2) $\mu = \mu_r \mu_0$ = Permeability(Henrys/mtr)
μ_r = Relative permeability of the material (Dimensionless)

μ_0 = free space permeability = $4\pi \times 10^{-7}$ Henrys/mtr

H = Magenetic fieldintensity AT/mtr N = Number of turns of thecoil

i = Current in thecoil (Amps) l = Length of the

coil (mtrs) from the above relations we can get ias

$i = H\, l/N = (1/\mu)(\Phi/A)\, l / N = (1/\mu)(\Phi/N)\, l / A = (1/\mu)(\Phi/N)\, [\, 2\pi r_T / \pi c r^2\,] = [\, 2 r_T\, \Phi / \mu\, c N r^2\,]$

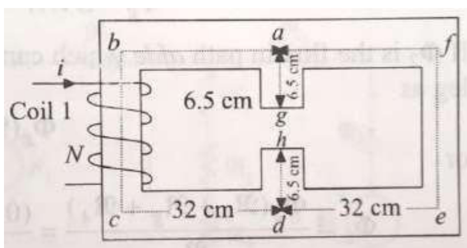
Where $r_T$is the radius of the toroid and r_C is the radius of cross section of the coil

Now we can calculate the currents in the two cases by substituting the respective values.

(a) Here $\mu = \mu_0$. Therefore $i = (2 \times 6 \times 10^{-2} \times 4 \times 10^{-4}) / (4\pi \times 10^{-7} \times 1000 \times 10^{-4}) = 380$ Amps
(b) Here $\mu = \mu_r\mu_0$. Therefore $i = (2 \times 6 \times 10^{-2} \times 4 \times 10^{-4}) / (4000 \times 4\pi \times 10^{-7} \times 1000 \times 10^{-4}) = 0.095$ Amps

Ex.2: (a) Draw the electrical equivalent circuit of the magnetic circuit shown in the figure below. The area of the core is 2x2 cm^2 .The length of the air gap is 1cm and lengths of the other limbs are shown in the figure. The relative permeability of the core is4000.

(b) Find the value of the current 'i' in the above example which produces a flux density of 1.2 Tesla in the air gap . The number f turns of the coil are5000.



Solution: (a)

To draw the equivalent circuit we have to find the Reluctances of the various flux

paths independently. The reluctance of the path *abcd* is given by: R1 = length of the

path *abcd*/$\mu_r\mu_0 A$

$= (32\times10^{-2}) / (4\pi \times 10^{-7} \times 4000 \times 4\times10^{-4}) = 1.59 \times 10^5$ AT/Wb

The reluctance of the path *afed*is equal to the reluctance of the path *abcd*since it has the same length, same permeability and same cross sectional area. Thus R1 = R2

Similarly the reluctance of the path *ag* (R3) is equal to that of the path *hd*(R4) and can be calculated as:
R3 = R4= $(6.5 \times 10^{-2}) / (4\pi \times 10^{-7} \times 4000 \times 4\times10^{-4}) = 0.32 \times 10^5$ AT/Wb

The reluctance of the air gap path *gh*RG can be calculated as : RG = length of the air gap path $gh/\mu_0 A$ (Here it is to be noted that μ is to be taken as μ_0 only and μ_r should not be included)

$R_G = (1 \times 10^{-2}) / (4\pi \times 10^{-7} \times 4 \times 10^{-4}) = 198.94 \times 10^5$ AT/Wb

MODULE II

Resonance and Three Phase Circuits

Three Phase Circuits Introduction:

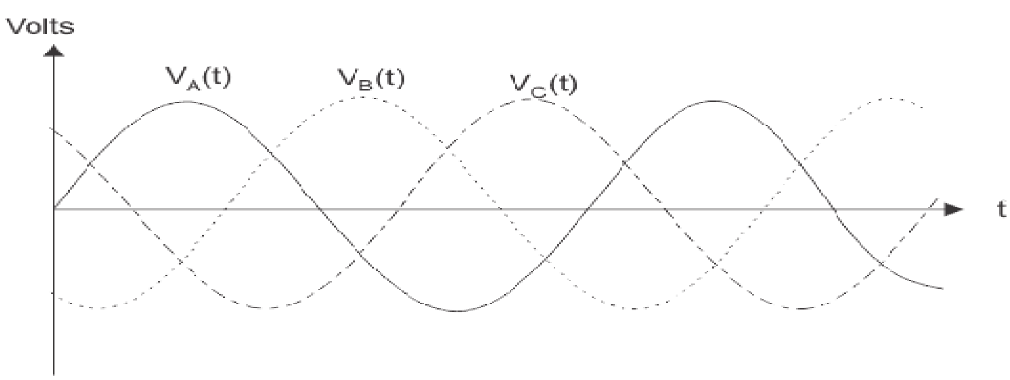
Three-phase systems are commonly used in generation, transmission and distribution of electric power. Power in a three-phase system is constant rather than pulsating and three-phase motors start and run much better than single-phase motors. A three-phase system is a generator-load pair in which the generator produces three sinusoidal voltages of equal amplitude and frequency but differing in phase by 120° from each other.

There are two types of system available in electric circuit, single phase and three phase system. In single phase circuit, there will be only one phase, i.e the current will flow through only one wire and there will be one return path called neutral line to complete the circuit. So in single phase minimum amount of power can be transported. Here the generating station and load station will also be single phase. This is an old system using from previous time.

In poly phase system, that more than one phase can be used for generating, transmitting and for load system. Three phase circuit is the polyp hase system where three phases are send together from the generator to the load. Each phase is having a phase difference of 120°, i.e 120° angle electrically. So from the total of 360°, three phases are equally divided into 120° each. The power in three phase system is continuous as all the three phases are involved in generating the total power. The sinusoidal waves for 3 phase system is shown below the three phases can be used as single phase each. So if the load is single phase, then one phase can be taken from the three phase circuit and the neutral can be used as ground to complete the circuit..

The phase voltages $v_a(t)$, $v_b(t)$ and $v_c(t)$ are as follows

$$v_a = V_m \cos\omega t$$
$$v_b = V_m \cos(\omega t - 120^\circ)$$
$$v_c = V_m \cos(\omega t - 240^\circ),$$

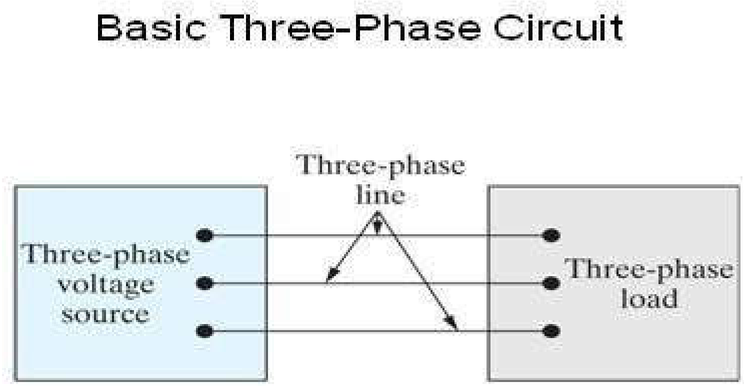


the corresponding phasors are

$V_a = V_m$

$V_b = V_m e^{-j120^\circ}$

$V_c = V_m e^{-j240^\circ}$



Basic three Phase Circuit

Advantages of Three Phase is preferred Over Single Phase

The three phase system can be used as three single phase line so it can act as three single phase system. The three phases generation and single phase generation is same in the generator except the arrangement of coil in the generator to get 120° phase difference. The conductor needed in three phase circuit is 75% that of conductor needed in single phase circuit. And also the instantaneous power in single phase system falls down to zero as in single phase we can see from the sinusoidal curve but in three phase system the net power from all the phases gives a continuous power to the load. the will have better and higher efficiency compared to the single phase system.

In three phase circuit, connections can be given in two types:

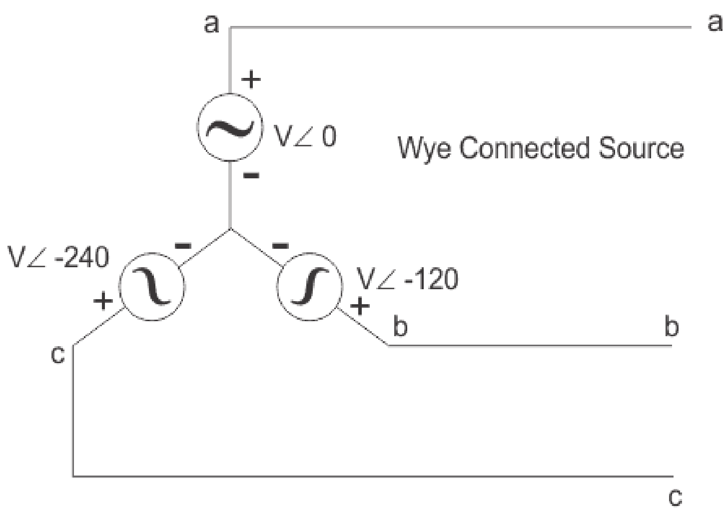
1. Star connection
2. Delta connection

STAR CONNECTION

In star connection, there is four wire, three wires are phase wire and fourth is neutral which is taken from the star point. Star connection is preferred for long distance power transmission because it is having the neutral point. In this we need to come to the concept of balanced and unbalanced current in power system.

When equal current will flow through all the three phases, then it is called as balanced current. And when the current will not be equal in any of the phase, then it is unbalanced current. In this case, during balanced condition there will be no current flowing through the neutral line and hence there is no use of the neutral terminal. But when there will be unbalanced current flowing in the three phase circuit, neutral is having a vital role. It will take the unbalanced current through to the ground and protect the transformer. Unbalanced current affects transformer and it may also cause damage to the transformer and for this star connection is preferred for long distance transmission.

The Star Connection



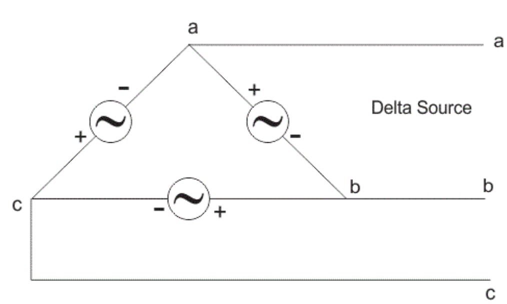
In star connection, the line voltage is √3 times of phase voltage. Line voltage is the voltage between two phases in three phase circuit and phase voltage is the voltage between one phase to the neutral line. And the current is same for both line and phase. It is shown as expression below

$$E_{Line} = \sqrt{3}E_{phase} \text{ } and \text{ } I_{Line} = I_{Phase}$$

Delta Connection

In delta connection, there are three wires alone and no neutral terminal is taken. Normally delta connection is preferred for short distance due to the problem of unbalanced current in the circuit. The figure is shown below for delta connection. In the load station, ground can be used as neutral path if required. In delta connection, the line voltage is same with that of phase voltage. And the line current is √3 times of phase current. It is shown as expression below,

$$E_{Line} = E_{phase} \text{ } and \text{ } I_{Line} = \sqrt{3}I_{Phase}$$



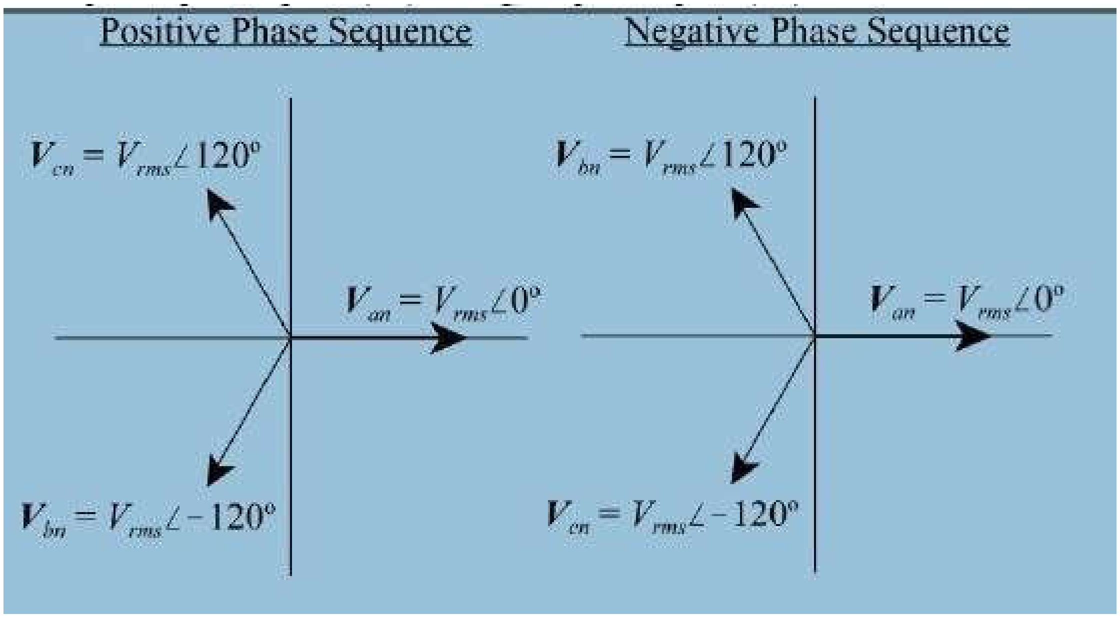
If we compare the line-to-neutral voltages with the line-to-line voltages, we find the following relationships,

Line-to-neutral voltages	Line-to-line voltages
$V_{an} = V_{rms} \angle 0°$	$V_{ab} = \sqrt{3} V_{rms} \angle 30°$
$V_{bn} = V_{rms} \angle -120°$	$V_{bc} = \sqrt{3} V_{rms} \angle -90°$
$V_{cn} = V_{rms} \angle 120°$	$V_{ca} = \sqrt{3} V_{rms} \angle 150°$

In three phase circuit, star and delta connection can be arranged in four different ways-

1. Star-Star connection
2. Star-Delta connection
3. Delta-Star connection
4. Delta-Delta connection

Phase Sequence



But the power is independent of the circuit arrangement of the three phase system. The net power in the circuit will be same in both star and delta connection. The power in three phase circuit can be calculated from the equation below,

$$P_{Total} = 3 \times E_{phase} \times I_{phase} \times PF$$

Since there is three phases, so the multiple of 3 is made in the normal power equation and the PF is power factor. Power factor is a very important factor in three phase system and sometimes due to certain error, it is corrected by using capacitors.

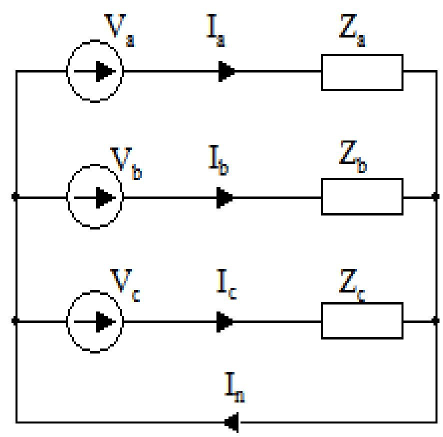
ANALYSIS OF BALANCED THREE PHASE CIRCUITS

In a balanced system, each of the three instantaneous voltages has equal amplitudes, but is separated from the other voltages by a phase angle of 120. The three voltages (or phases) are typically labeled a, b and c. The common reference point for the three phase voltages is designated as the neutral connection and is labeled .
A three-phase system is shown in Fig. In a special case all impedances are identical

$$Z_a = Z_b = Z_c = Z$$

Such a load is called a balanced load and is described by equations

$$I_a = \frac{V_a}{Z} \qquad I_b = \frac{V_b}{Z} \qquad I_c = \frac{V_c}{Z}.$$



Using KCL, we have,

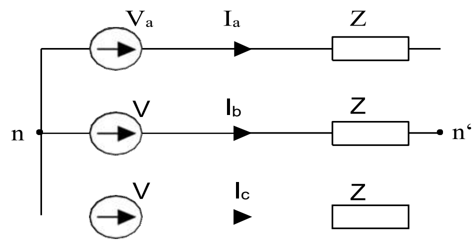
$$V_a + V_b + V_c = V_m \left(1 + e^{-j120^\circ} + e^{-j240^\circ}\right)$$

$$= V_m\left(1 + \cos 120^\circ - j\sin 120^\circ + \cos 240^\circ - j\sin 240^\circ\right) = V_m\left(1 - \frac{1}{2} - j\frac{\sqrt{3}}{2} - \frac{1}{2} + j\frac{\sqrt{3}}{2}\right) = 0.$$

$$I_n = I_a + I_b + I_c = \frac{1}{Z}\left(V_a + V_b + V_c\right), \quad (4)$$

From the above result, we obtain $I_n = 0$.

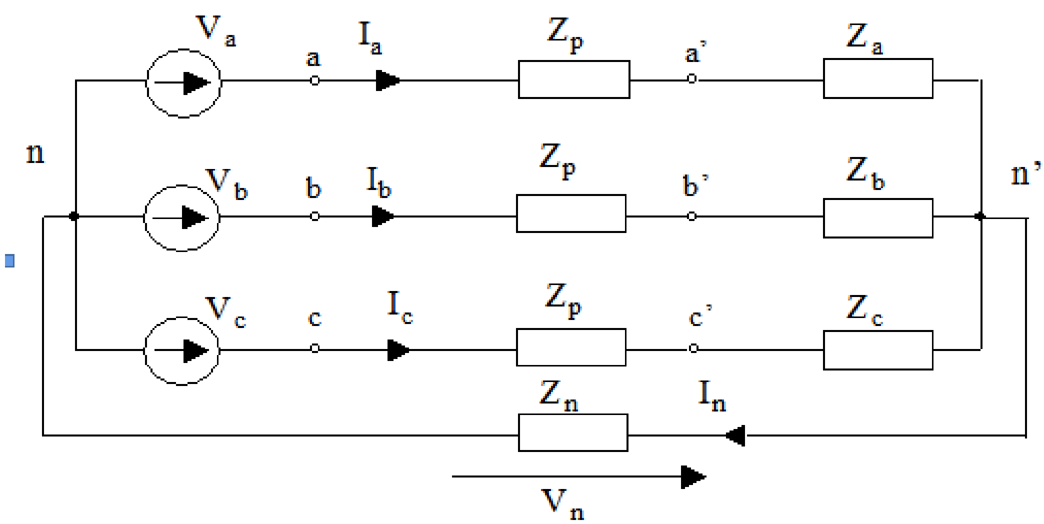
Since the current flowing though the fourth wire is zero, the wire can be removed



ANALYSIS OF UNBALANCED LOADS

Three-phase systems deliver power in enormous amounts to single-phase loads such as lamps, heaters, air-conditioners, and small motors. It is the responsibility of the power systems engineer to distribute these loads equally among the three-phases to maintain the demand for power fairly balanced at all times. While good balance can be achieved on large power systems, individual loads on smaller systems are generally unbalanced and must be analyzed as unbalanced three phase systems.

When the three phases of the load are not identical, an unbalanced system is produced. An unbalanced Y-connected system is shown in Fig.1. The system of Fig.1 contains perfectly conducting wires connecting the source to the load. Now we consider a more realistic case where the wires are represented by impedances Z_p and the neutral wire connecting n and n‘ is represented by impedance Z_n



the node n as the datum, we express the currents I_a, I_b, I_c and I_n in terms of the node voltage V_n

$$I_a = \frac{V_a - V_n}{Z_a + Z_p}$$

$$I_b = \frac{V_b - V_n}{Z_b + Z_p}$$

$$I_c = \frac{V_c - V_n}{Z_c + Z_p}$$

$$I_n = \frac{V_n}{Z_n}$$

The node equation is

$$\frac{V_n}{Z_n} - \frac{V_a - V_n}{Z_a + Z_p} - \frac{V_b - V_n}{Z_b + Z_p} - \frac{V_c - V_n}{Z_c + Z_p} = 0$$

And $$V_n = \frac{\dfrac{V_a}{Z_a + Z_p} + \dfrac{V_b}{Z_b + Z_p} + \dfrac{V_c}{Z_c + Z_p}}{\dfrac{1}{Z_n} + \dfrac{1}{Z_a + Z_p} + \dfrac{1}{Z_b + Z_p} + \dfrac{1}{Z_c + Z_p}}$$

Power in three-phase circuits

In the balanced systems, the average power consumed by each load branch is the same and given by

$$\tilde{P}_{av} = V_{eff} I_{eff} \cos\theta$$

where V_{eff} is the effective value of the phase voltage, I_{eff} is the effective value of the phase current and θ is the angle of the impedance. The total average power consumed by the load is the sum of those consumed by each branch, hence, we have

$$P_{av} = 3\tilde{P}_{av} = 3 V_{eff} I_{eff} \cos\theta$$

In the balanced Y systems, the phase current has the same amplitude as the line current $I_{eff} = |I_{eff}|_L$, whereas the line voltage has the effective value $|V_{eff}|_L$ Which is $\sqrt{3}$ times greater than the effective value of the phase voltage, $|V_{eff}|_L = \sqrt{3} V_{eff}$. Hence, using (22), we obtain

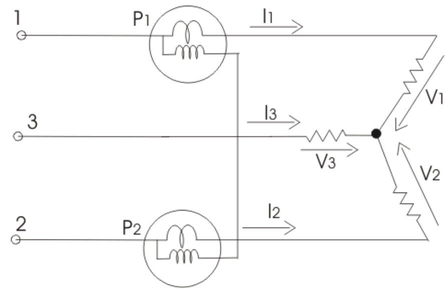
Measurement of Three Phase Power by Two Wattmeter's Method

In this method we have two types of connections

(a) Star connection of loads

(b) Delta connection of loads.

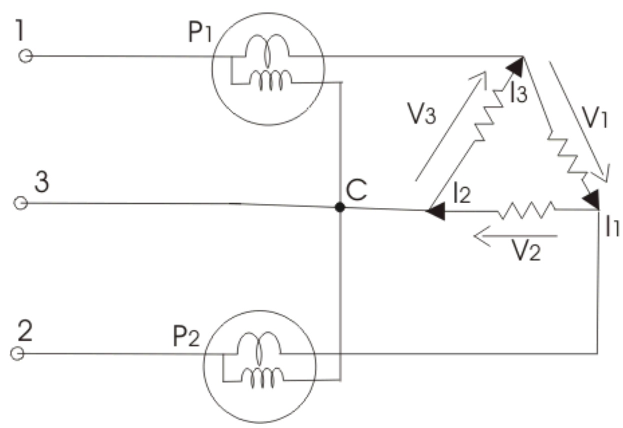
When the star connected load, the diagram is shown in below-



For star connected load clearly the reading of wattmeter one is product phase current and voltagedifference (V_2-V_3). Similarly the reading of wattmeter two is the product of phase current and the voltage difference (V_2-V_3). Thus the total power of the circuit is sum of the reading of both the wattmeter's.Mathematically we can write

$$P = P_1 + + P_2 = I_1(V_1 + V_2) + I_2(V_2 - V_3)$$

But we have I1+I2+I3=0, hence putting the value of I1+I2=-I3. We get total power as V1I1+V2I2+V3I3.
For delta connected load, the diagram is shown in below



The reading of wattmeter one can be written as

$$P_1 = -V_3(I_1 - I_3)$$

And reading of wattmeter two is

$$P_2 = -V_2(I_2 - I_1)$$

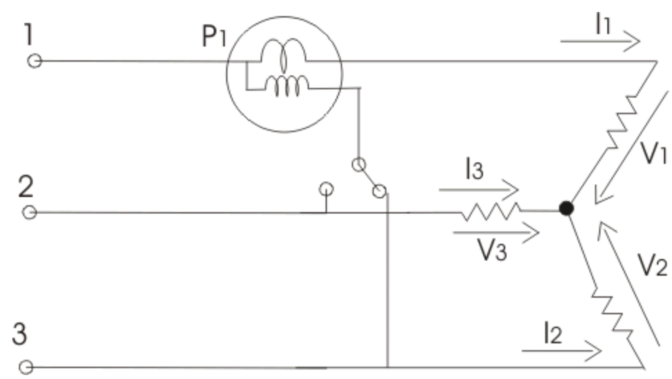
$$Total\ power\ is\ P = P_1 + P_2 = V_2 I_2 + V_3 I_3 - I_1(V_2 + V_3)$$

But $V_1+V_2+V_3=0$, hence expression for total power will reduce to $V_1I1+V_2I_2+V_3I_3$.

13.16 Measurement of Three Phase Power by One Wattmeter Method

Limitation of this method is that it cannot be applied on unbalanced load. So under this condition we have $I_1=I_2=I_3=I$ and $V_1=V_2=V_3=V$.

Diagram is shown below:



Two switches are given which are marked as 1-3 and 1-2, by closing the switch 1-3 we get reading of wattmeter as

$$P_1 = V_{13} I_1 \cos(30 - \phi) = \sqrt{3} \times VI \cos(30 - \phi)$$

Similarly the reading of wattmeter when switch 1-2 is closed is

$$P_2 = V_{12} I_1 \cos(30 + \phi) = \sqrt{3} \times VI \cos(30 + \phi)$$

$$Total\ power\ is\ P_1 + P_2 = 3VI\cos\phi$$

Locus diagrams

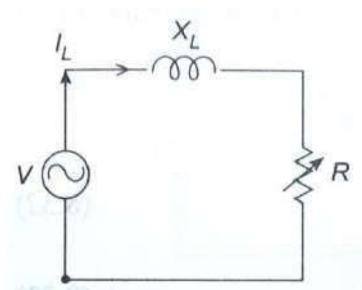
Introduction: In AC electrical circuits the magnitude and phase of the current vector depends upon the values of R,L&C when the applied voltage and frequency are kept constant. The path traced by the terminus (tip) of the current vector when the parameters R,L&C are varied is called the current Locus diagram . Locus diagrams are useful in studying and understanding the behavior of the RLC circuits when one of these parameters is varied keeping voltage and frequency constant.

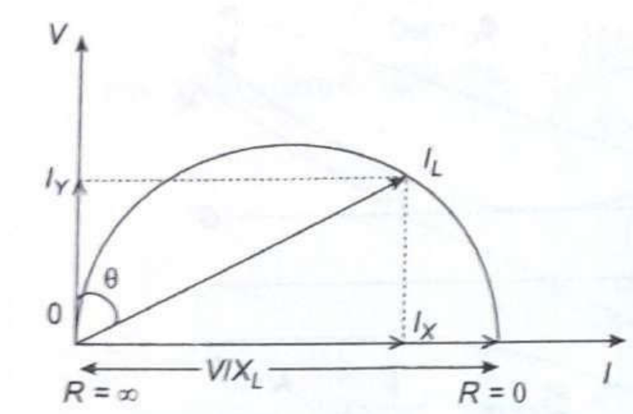
In this unit,Locus diagrams are developed and explained for series RC,RL circuits and Parallel LC circuits along with their internal resistances when the parameters R,L and C are varied.

The term circle diagram identifies locus plots that are either circular or semicircular. The defining equations of such circle diagrams are also derived in this unit for series RC and RL diagrams.
In both series RC,RL circuits and parallel LC circuits resistances are taken to be in series with L and C to highlight the fact that all practical L and C components will have at least a small value of internal resistance.

Series RL circuit with varying Resistance R:

Refer to the series RL circuit shown in the figure (a) below with constant X_L and varying R. The current I_L lags behind the applied voltage V by a phase angle $\Theta = \tan^{-1}(X_L/R)$ for a given value of R as shown in the figure (b) below. When R=0 we can see that the current is maximum equal to V/X_L and lies along the I axis with phase angle equal to 90^0. When R is increased from zero to infinity the current gradually reduces from V/X_Lto0andphaseanglealsoreducesfrom$90^0$to
0^0.Ascanbeseenfromthefigure,thetipofthecurrentvectortracesthepathofasemicircle
with its diameter along the +ve I axis.





Fig(a): Series RL circuit with **Fig(b): Locus of current vector I_L with variation**
of R Varying Resistance R

The related equations are:
$I_L = V/Z$ Sin $\Theta = X_L/Z$ or $Z = X_L/ \text{Sin}\,\Theta$ and Cos Θ
$= R / Z$ Therefore $I_L = (V/X_L)\,\text{Sin}\Theta$
For constant V and X_L the above expression for I_L is the polar equation of a circle with diameter (V/X_L) as shown in the figure above.

Circle equation for the RL circuit: (with fixed reactance and variable Resistance):

The X and Y coordinates of the current
$I_L$are $I_X = I_L\text{Sin}\Theta$ $\quad I_Y = I_L\,\text{Cos}\Theta$
From the relations given above and earlier we get

$$I_X = (V/Z)(X_L/Z) = VX_L/Z^2 \text{-----------------------------} (1)$$

and

$$I_Y = (V/Z)(R/Z) = V\,R/Z^2 \text{-----------------------------} (2)$$

Squaring and adding the above two equations we get

$$I_X^2 + I_Y^2 = V^2(X_L^2+R^2) / Z^4 = (V^2Z^2) / Z^4 = V^2/Z^2 \text{-------------} (3)$$

From equation (1) above we have $Z^2 = V X_L / I_X$ and substituting this in the above equation (3) we get:

$$I_X^2 + I_Y^2 = V^2 / (V X_L / I_X) = (V/X_L)x I$$

$$\text{or } I_X^2 + I_Y^2 - (V/X_L)x I = 0$$

Adding $(V/2X_L)^2$ to both sides ,the above equation can be written as

$$[I_X - V/2X_L]^2 + I_Y^2 = (V/2X_L)^2 \text{------------------------------------} (4)$$

Equation (4) above represents a circle with a radius of $(V/2X_L)$ and with it's coordinates of the centre as $(V/2X_L , 0)$

Series RC circuit with varying Resistance R:

Refer to the series RC circuit shown in the figure (a) below with constant X_C and varying R. The current I_C leads the applied voltage V by a phase angle $\Theta = \tan^{-1}(X_C/R)$ for a given value of R as shown in the figure (b) below. When R=0 we can see that the current is maximum equal to $-V/X_C$ and lies along the negative I axis with phase angle equal to -90^0. When R is increased from zero to infinity the current gradually reduces from $-V/X_C$ to 0 and phase angle also reduces from -90^0 to 0^0. As can be seen from the figure, the tip of the current vector traces the path of a semicircle but now with its diameter along the negative Iaxis.

Circle equation for the RC circuit: (with fixed reactance and variable Resistance):

In the same way as we got for the Series RL circuit with varying resistance we can get the circle equation for an RC circuit with varying resistance as :

$$[I_X + V/2X_C]^2 + I_Y^2 = (V/2X_C)^2$$

Whose coordinates of the centre are $(-V/2X_C , 0)$ and radius equal to $V/2X_C$

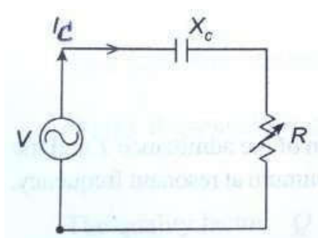


Fig: Series RCcircuitwith VaryingResistanceR

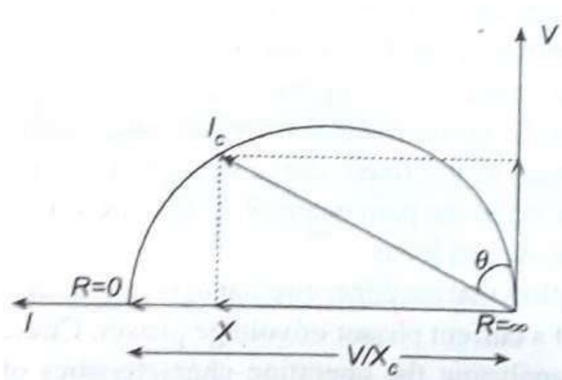
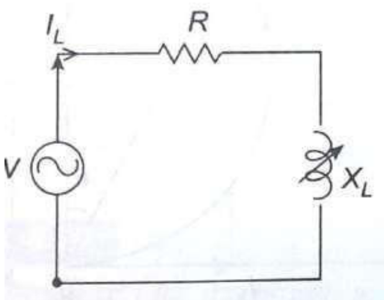


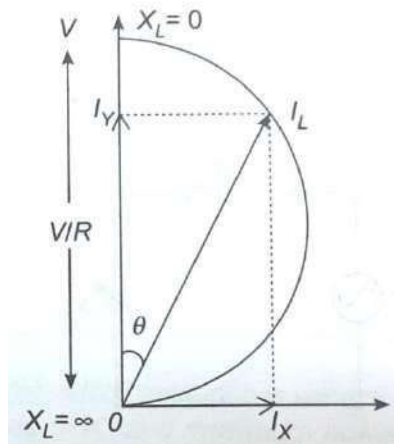
Fig: Locus of current vectorI$_C$ with variation ofR

Series RL circuit with varying Reactance XL:

Refer to the series RL circuit shown in the figure (a) below with constant R and varying X_L. The current I_L lags behind the applied voltage V by a phase angle $\Theta = \tan^{-1}(X_L/R)$ for a given value of

R as shown in the figure (b) below. When X_L =0 we can see that the current is maximum equal to V/R and lies along the +ve V axis with phase angle equal to 0^0. When X_L is increased from zero to infinity the current gradually reduces from V/R to 0 and phase angle increases from 0^0 to 90^0. As can be seen from the figure, the tip of the current vector traces the path of a semicircle with its diameter along the +ve V axis and on to its rightside.

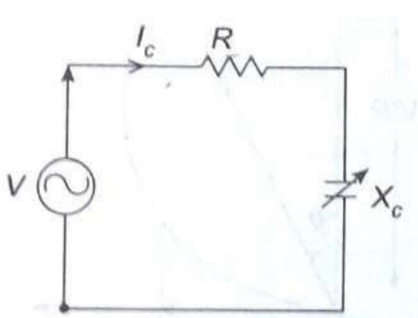


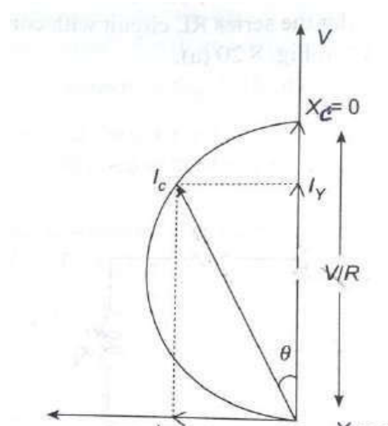


Fig(a): Series RL circuit with varying X_L Fig(b) : Locus of current vector I_L with variation of X_L

Series RC circuit with varying Reactance XC:

Refer to the series RC circuit shown in the figure (a) below with constant R and varying X_C. The current I_C leads the applied voltage V by a phase angle $\Theta = \tan^{-1}(X_C/R)$ for a given value of R as shown in the figure (b) below. When X_C =0 we can see that the current is maximum equal to V/R and lies along the V axis with phase angle equal to 0^0. When X_C is increased from zero to infinity the current gradually reduces from V/R to 0 and phase angle increases from 0^0 to -90^0. As can be seen from the figure, the tip of the current vector traces the path of a semicircle with its diameter along the +ve V axis but now on to its leftside.





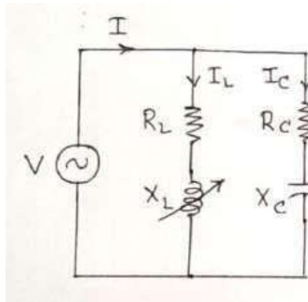
Fig(a): Series RC circuit with varying X_C Fig(b): Locus of current vector I_C with variation of X_C

Parallel LC circuits:

Parallel LC circuit along with its internal resistances as shown in the figures below is considered here for drawing the locus diagrams. As can be seen, there are two branch currents I_C and I_L

along with the total current I. Locus diagrams of the current I_L or I_C (depending on which arm is varied)and the total current I are drawn by varying R_L, R_C , X_L and X_C one by one.

Varying X_L:



Fig(a): parallel LC circuit with Internal Resistances R_L and R_C in series with L (Variable) and C (fixed)respectively.

The current I_C through the capacitor is constant since R_C and C are fixed and it leads the voltage vector OV by an angle $\Theta_C = \tan^{-1}(X_C/R_C)$ as shown in the figure (b). The current I_L through the inductance is the vector OI_L . It's amplitude is maximum and equal to V/R_L when X_L is zero and it is in phase with the applied voltage V. When X_L is increased from zero to infinity it's amplitude decreases to zero and phase will be lagging the voltage by 90^0. In between, the phase angle will be lagging the voltage V by an angle $\Theta_L = \tan^{-1}(X_L/R_L)$. The locus of the current vector I_L is a semicircle with a diameter of length equal to V/R_L. Note that this is the same locus what we got earlier for the series RL circuit with X_L varying except that here V is shown horizontally.
Now, to get the locus of the total current vector OI we have to add vectorially the currents I_C and I_L . We know that to get the sum of two vectors geometrically we have to place one of the vectors staring point (we will take varying amplitude vector I_L)at the tip of the other vector (we will take constant amplitude vector I_C)and then join the start of fixed vector I_C to the end of varying vector I_L. Using this principle we can get the locus of the total current vector OI by shifting the I_L semicircle starting point O to the end of current vector OI_C keeping the two diameters parallel. The resulting semicircle I_CIB_T shown in the figure in dotted lines is the locus of the total current vectorOI.

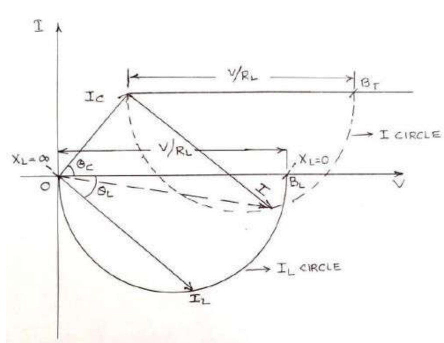


Fig (b): Locus of current vector I in Parallel LC circuit when X_L is varied from 0 to

Varying XC:

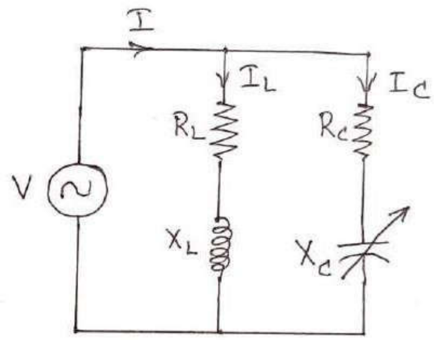
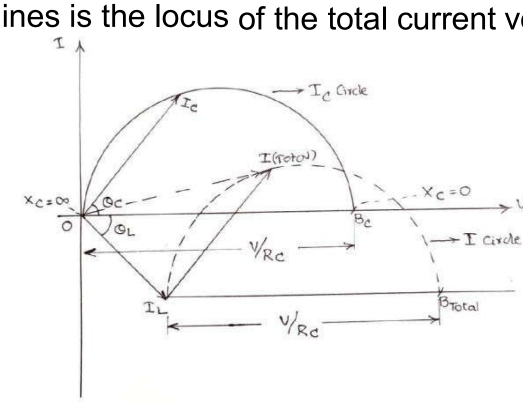


Fig.(a) parallel LC circuit with Internal Resistances R_L and R_C in series with L (fixed) and C (Variable)respectively.

The current I_L through the inductor is constant since R_L and L are fixed and it lags the voltage vector OV by an angle $\Theta_L = \tan^{-1}(X_L/R_L)$ as shown in the figure (b). The current I_C through the capacitance is the vector OI_C . It's amplitude is maximum and equal to V/R_C when X_C is zero and it is in phase with the applied voltage V. When X_C is increased from zero to infinity it's amplitude decreases to zero and phase will be leading the voltage by 90^0. In between, the phase angle will be leading the voltage V by an angle $\Theta_C = \tan^{-1}(X_C/R_C)$. The locus of the current vector I_C is a semicircle with a diameter of length equal to V/R_Cas shown in the figure below. Note that this is the same locus what we got earlier for the series RC circuit with X_C varying except that here V is shown horizontally.

Now, to get the locus of the total current vector OI we have to add vectorially the currents I_C and I_L . We know that to get the sum of two vectors geometrically we have to place one of the vectors staring point (we will take varying amplitude vector I_C)at the tip of the other vector (we will take constant amplitude vector I_L) and then join the start of the fixed vector I_L to the end of varying vector I_C. Using this principle we can get the locus of the total current vector OI by shifting the I_C semicircle starting point O to the end of current vector OI_L keeping the two diameters parallel. The resulting semicircle I_LIB_T shown in the figure in dotted lines is the locus of the total current vectorOI.



Fig(b) : Locus of current vector I in Parallel LC circuit when X_C is varied from 0 to ∞

Varying RL:

The current I_C through the capacitor is constant since R_C and C are fixed and it leads the voltage vector OV by an angle $\Theta_C = \tan^{-1}(X_C/R_C)$ as shown in the figure (b). The current I_L through the inductance is the vector OI_L . It's amplitude is maximum and equal to V/X_L when R_L is zero. Its phase will be lagging the voltage by 90^0. When R_L is increased from zero to infinity it's amplitude decreases to zero and it is in phase with the applied voltage V. In between, the phase angle will be lagging the voltage V by an angle $\Theta_L = \tan^{-1}(X_L/R_L)$. The locus of the current vector I_L is a semicircle with a diameter of length equal to V/R_L. Note that this is the same locus what we got earlier for the series RL circuit with R varying except that here V is shown horizontally.

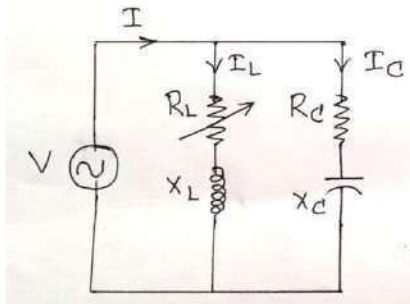
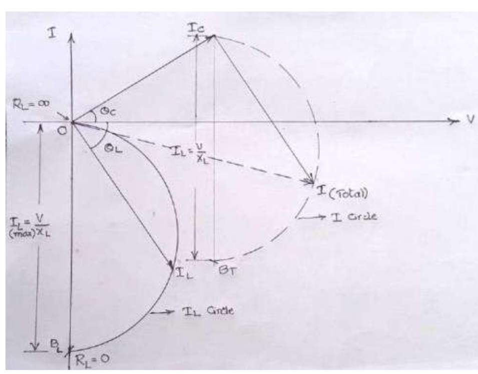


Fig.(a) parallel LC circuit with Internal Resistances R_L (Variable) and R_C (fixed) in series with L and C respectively.

Now, to get the locus of the total current vector OI we have to add vectorially the currents I_C and I_L . We know that to get the sum of two vectors geometrically we have to place one of the vectors staring point (we will take varying amplitude vector I_L)at the tip of the other vector (we will take constant amplitude vector I_C)and then join the start of fixed vector I_C to the end of varying vector I_L. Using this principle we can get the locus of the total current vector OI by shifting the I_L semicircle starting point O to the end of current vector OI_C keeping the two diameters parallel. The resulting semicircle I_CIB_T shown in the figure in dotted lines is the locus of the total current vectorOI.



Fig(b) : Locus of current vector I in Parallel LC circuit when R_L is varied from 0 to

Varying RC:

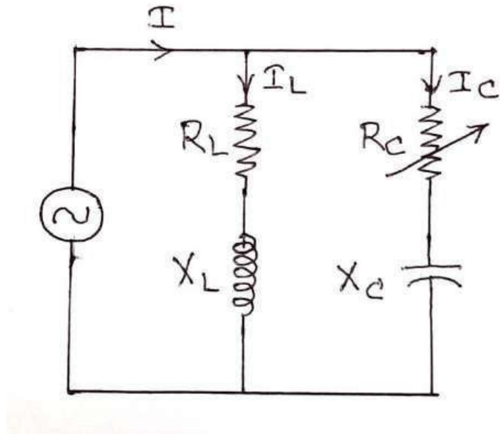
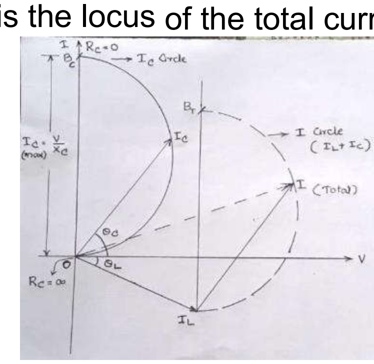


Fig.(a) parallel LC circuit with Internal Resistances R_L(fixed) andR_C(Variable) in series with L and Crespectively.

The current I_L through the inductor is constant since R_L and L are fixed and it lags the voltage vector OV by an angle $\Theta_L = \tan^{-1}(X_L/R_L)$ as shown in the figure (b). The current I_C through the capacitance is the vector OI_C . It's amplitude is maximum and equal to V/X_C when R_C is zero and its phase will be leading the voltage by 90^0 . When R_C is increased from zero to infinity it's amplitude decreases to zero and it will be in phase with the applied voltage V. In between, the phase angle will be leading the voltage V by an angle $\Theta_C = \tan^{-1}(X_C/R_C)$. The locus of the current vector I_C is a semicircle with a diameter of length equal to V/X_C as shown in the figure below. Note that this is the same locus what we got earlier for the series RC circuit with R varying except that here V is shown horizontally.

Now, to get the locus of the total current vector OI we have to add vectorially the currents I_C and I_L . We know that to get the sum of two vectors geometrically we have to place one of the vectors staring point (we will take varying amplitude vector I_C)at the tip of the other vector (we will take constant amplitude vector I_L) and then join the start of the fixed vector I_L to the end of varying vector I_C. Using this principle we can get the locus of the total current vector OI by shifting the I_C semicircle starting point O to the end of current vector OI_L keeping the two diameters parallel. The resulting semicircle I_LIB_T shown in the figure in dotted lines is the locus of the total current vectorOI.



Fig(b) : Locus of current vector I in Parallel LC circuit when R_C is varied from 0 to

Resonance:

Series RLC circuit:

The impedance of the series RLC circuit shown in the figure below and the current I through the circuit are given by :

$$Z = R + j\omega L + 1/j\omega C = R + j(\omega L - 1/\omega C) \quad I = Vs/Z$$

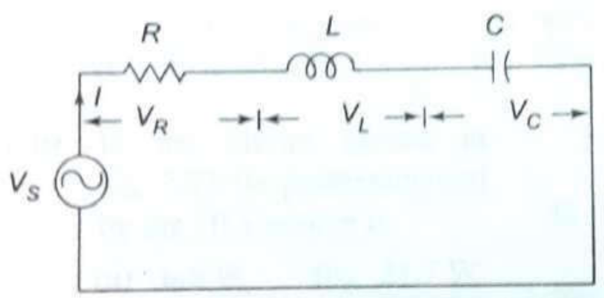


Fig: Series RLC circuit

The circuit is said to be in resonance when the Inductive reactance is equal to the Capacitive reactance. i.e. $\mathbf{X_L = X_C}$ **or** $\mathbf{\omega L = 1/\omega C}$**.** (i.e. Imaginary of the impedance is zero) The frequency

at which the resonance occurs is called resonant frequency. In the resonant condition when $X_L = X_C$ they cancel with each other since they are in phase opposition(180^0 out of phase) and net impedance of the circuit is purely resistive.In this condition the magnitudes of voltages across the Capacitance and the Inductance are also equal to each other but again since they are of opposite polarity they cancel with each other and the entire applied voltage appears across the Resistance alone**.**

Solving for the resonant frequency from the above condition of Resonance **:** $\mathbf{\omega L = 1/\omega C}$

$$2\pi f_r L = 1/2\pi f_r C$$

$$f_r^2 = 1/4\pi^2 LC \quad \text{and} \quad f_r = 1/2\pi\sqrt{LC}$$

In a series RLC circuit, resonance may be produced by varying L or C at a fixed frequency or by varying frequency at fixed L and C.

Reactances, Impedance and Resistance of a Series RLC circuit as a function of frequency:

From the expressions for the Inductive and capacitive reactances we can see that when the frequency is zero, capacitance acts as an open circuit and Inductance as a short circuit. Similarly when the frequency is infinity inductance acts as an open circuit and the capacitance acts as a short circuit. The variation of Inductive and capacitive reactances along with Resistance R and the Total Impedance are shown plotted in the figure below.

As can be seen, when the frequency increases from zero to ∞ Inductive reactance X_L (directly proportional to ω) increases from zero to ∞ and Capacitive reactance X_C (inversely proportional to ω) decreases from −∞ to zero. Whereas, the Impedance decreases from ∞ to Pure Resistance R as the frequency increases from zero to f_r (as capacitive reactance reduces from

−∞ and becomes equal to Inductive reactance) and then increases from R to ∞ as the frequency increases from f_r to ∞ (as inductive reactance increases from its value at resonant frequency to ∞)

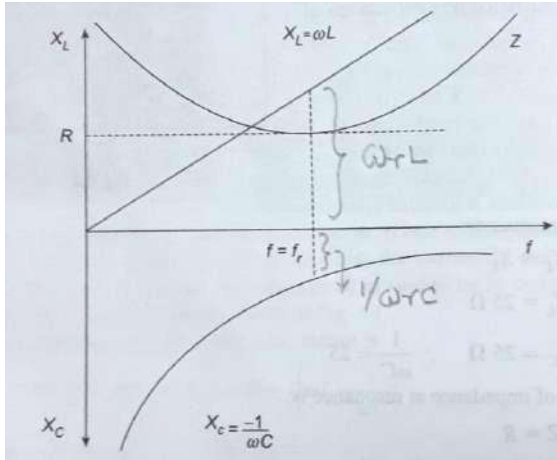


Fig : Reactance and Impedance plots of a Series RLC circuit

Phase angle of a Series RLC circuit as a function of frequency:

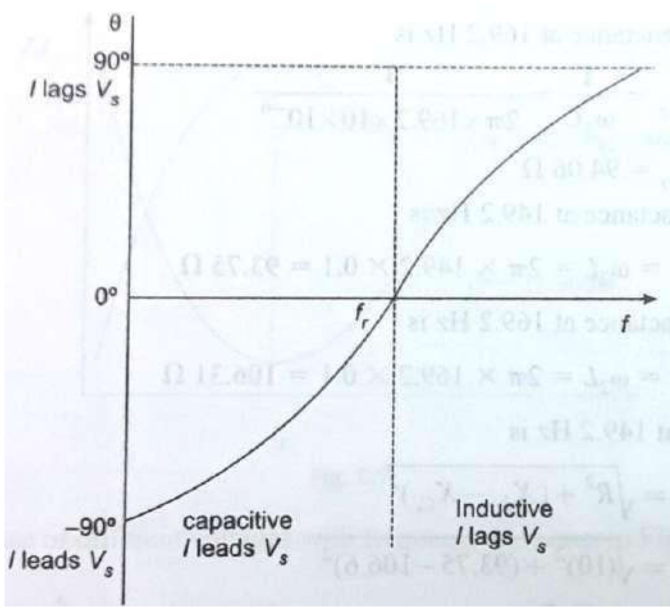


Fig : Phase plot of a Series RLC circuit

The following points can be seen from the Phase angle plot shown in the figure above:

- At frequencies below the resonant frequency capacitive reactance is higher than the inductive reactance and hence the phase angle of the current leads the voltage.
- As frequency increases from zero to $\mathbf{f_r}$ the phase angle changes from -90^0 to zero.

- At frequencies above the resonant frequency inductive reactance is higher than the capacitive reactance and hence the phase angle of the current lags the voltage.
- As frequency increases from **f_r** and approaches ∞, the phase angle increases from zero and approaches 90^0

Band width of a Series RLC circuit:

The band width of a circuit is defined as the Range of frequencies between which the output power is half of or 3 db less than the output power at the resonant frequency. These frequencies are called the cutoff frequencies, 3db points or half power points. But when we consider the output voltage or current, the range of frequencies between which the output voltage or current falls to 0.707 times of the value at the resonant frequency is called the Bandwidth BW. This is because voltage/current are related to power by a factor of √ 2 and when we are consider √ 2 times less it becomes 0.707. But still these frequencies are called as cutoff frequencies, 3db points or half power points. The lower end frequency is called lower cutoff frequency and the higher end frequency is called upper cutoff frequency.

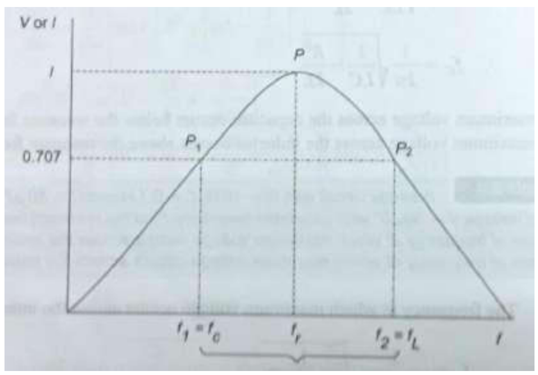


Fig: Plot showing the cutoff frequencies and Bandwidth of a series RLC circuit

Derivation of an expression for the BW of a series RLC circuit:

We know that BW = $f_2 - f_1$ Hz

If the current at points P_1 and P_2 are 0.707 (1/√ 2) times that of I_{max} (current at the resonant frequency) then the Impedance of the circuit at points P_1 and P_2 is √ 2 R (i.e. √ 2 times the impedance at f_r)

But Impedance at point P_1 is given by: $Z = \sqrt{R^2 + (1/\omega_1 C - \omega_1 L)^2}$ and equating this to √ 2 R we get: $(1/\omega_1 C) - \omega_1 L = R$ -------------- (1)

Similarly Impedance at point P_2 is given by: $Z = \sqrt{R^2 + (\omega_2 L - 1/\omega_2 C)^2}$ and equating this to √ 2 R we get: $\omega_2 L - (1/\omega_2 C) = R$ ------ (2)

Equating the above equations (1) and (2) we get:

$$1/\omega_1 C - \omega_1 L = \omega_2 L - 1/\omega_2 C$$

Rearranging we get $L(\omega_1+\omega_2) = 1/C\,[(\omega_1+\omega_2)/\omega_1\omega_2]$ i.e $\omega_1\omega_2 = 1/LC$

But we already know that for a series RLC circuit the resonant frequency is given by $\omega_r^2 = 1/LC$ Therefore: $\omega_1\omega_2 = \omega_r^2$ ---- (3) and $1/C = \omega_r^2 L$ ------ (4)

Next adding the above equations (1) and (2) we get:

$$1/\omega_1 C - \omega_1 L + \omega_2 L - 1/\omega_2 C = 2R \quad (\omega_2 - \omega_1)L + (1/\omega_1 C - 1/\omega_2 C) = 2R$$

$$(\omega_2 - \omega_1)L + 1/C[(\omega_2 - \omega_1)/\omega_1\omega_2) = 2R \quad \text{------} \quad (5)$$

Using the values of $\omega_1\omega_2$ and 1/C from equations (3) and (4) above into equation (5) above we get: $(\omega_2 - \omega_1)L + \omega^2 L\,[(\omega_2 - \omega_1)/\omega^2) = 2R$

i.e. $2L(\omega_2 - \omega_1) = 2R$ i.e. $(\omega_2 - \omega_1) = R/L$ and $(f_2 - f_1) = R/2\pi L$ ----- (6)

Or finally Band width $BW = R/2\pi L$ ----- (7)

Since f_r lies in the centre of the lower and upper cutoff frequencies f_1 and f_2 using the above equation (6) we can get:

$$\mathbf{f_1 = f_r - R/4\pi L} \quad \text{------} \quad (8)$$

$$\mathbf{f_2 = f_r + R/4\pi L} \quad \text{------} \quad (9)$$

Further by dividing the equation(6)a ov by $\mathbf{f_r}$ on both sides we get another **impor** t Relation: $(f_2 - f_1) / f_r = R/2\pi f_r L$ or $BW / f_r = R/2\pi f_r L$ ----------------(10)

Here an important property of a coil i.e. Q factor or figure of merit is defined as the ratio of the reactance to the resistance of acoil.

$$Q = 2\pi f_r L/R \text{...} (11)$$

Now using the relation (11) we can rewrite the relation (10) as

$$Q = f_r/BW \text{..} (12)$$

Quality factor of a series RLC circuit:

The quality factor of a series RLC circuit is defined as:

Q = Reactive power in Inductor (or Capacitor) at resonance / Average power at Resonance

Reactive power in Inductor at resonance = I^2X_L
Reactive power in Capacitor at resonance = I^2X_C
Average power at Resonance $= I^2R$

Here the power is expressed in the form I^2X (not as V^2/X) since I is common through R.L and C in the series RLC circuit and it gets cancelled during thes implification.

Therefore $Q = I^2X_L / I^2R = I^2X_C / I^2R$

i.e. $Q = X_L / R = \omega_r L/ R$ --------------------------------(1)

Or $Q = X_C / R = 1/\omega_r RC$ ....(2)

From these two relations we can also define Q factor as :

Q = Inductive (or Capacitive) reactance at resonance / Resistance

Substituting the value of $\omega_r = 1/\sqrt{LC}$ in the expressions (1) or (2) for Q above we can get the value of Q in terms of R, L,C as below.

$$Q = (1/\sqrt{LC})L/R = (1/R)(\sqrt{L/C})$$

Selectivity:

Selectivity of a series RLC circuit indicates how well the given circuit responds to a given resonant frequency and how well it rejects all other frequencies. i.e. the selectivity is directly proportional to Q factor. A circuit with a good selectivity (or a high Q factor) will have maximum gain at the resonant frequency and will have minimum gain at other frequencies .i.e. it will have very low band width. This is illustrated in the figure below.

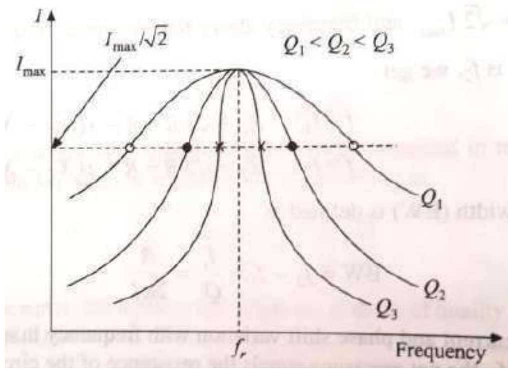


Fig: Effect of quality factor on bandwidth Voltage Magnification at resonance:

At resonance the voltages across the Inductance and capacitance are much larger than the applied voltage in a series RLC circuit and this is called voltage magnification at Resonance. The voltage magnification is equal to the Q factor of the circuit. This is proven below.

If we take the voltage applied to the circuit as V and the current through the circuit at resonance as I then

The voltage across the inductance L is: $V_L = IX_L = (V/R)\ \omega_r L$

and The voltage across the capacitance C is: $V_C = IX_C = V/R\ \omega_r C$

But we know that the Q of a series RLC circuit = $\omega_r L/R = 1/R\ \omega_r C$

Using these relations in the expressions for V_L and V_C given above we get $V_L = VQ$ and $V_C = VQ$

The ratio of voltage across the Inductor or capacitor at resonance to the applied voltage in a series RLC circuit is called Voltage magnification and is given by

$$\text{Magnification} = Q = V_L/V \text{ or } V_C/V$$

Important points In Series RLC circuit at resonant frequency:

- The impedance of the circuit becomes purely resistive and minimum i.e Z =R
- The current in the circuit becomes maximum
- The magnitudes of the capacitive Reactance and Inductive Reactance becomes equal
- The voltage across the Capacitor becomes equal to the voltage across the Inductor at resonance and is Q times higher than the voltage across the resistor

Bandwidth and Q factor of a Parallel RLC circuit:

Parallel RLC circuit is shown in the figure below. For finding out the **BW** and **Q** factor of a parallel RLC circuit, since it is easier we will work with Admittance , Conductance and Susceptance instead of Impedance ,Resistance and Reactance like in series RLC circuit.

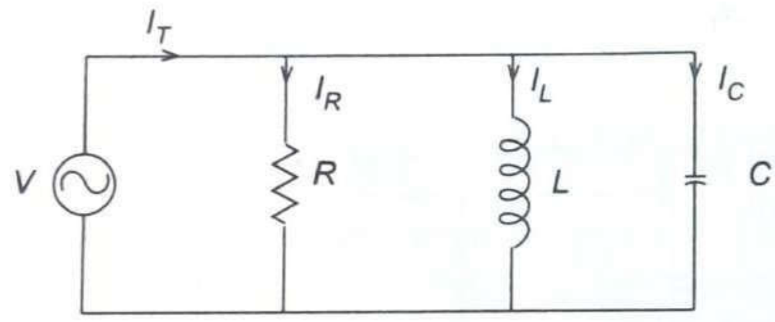


Fig: Parallel RLC circuit

Then we have the relation: $\mathbf{Y = 1/Z = 1/R + 1/j\omega L + j\omega C = 1/R + j\,(\omega C - 1/\omega L)}$

For the parallel RLC circuit also, at resonance, the imaginary part of the Admittance is zero and hence the frequency at which resonance occurs is given by: $\omega_r C - 1/\omega_r L = 0$. From this we get: $\omega_r C = 1/\omega_r L$ and $\omega_r = 1/\sqrt{LC}$
Which is the same value for ω_r as what we got for the series RLC circuit.

At resonance when the imaginary part of the admittance is zero the admittance becomes minimum.(i.e Impedance becomes maximum as against Impedance becoming minimum in series RLC circuit) i.e. Current becomes minimum in the parallel RLC circuit at resonance (as against current becoming maximum in series RLC circuit) and increases on either side of the resonant frequency as shown in the figure below.

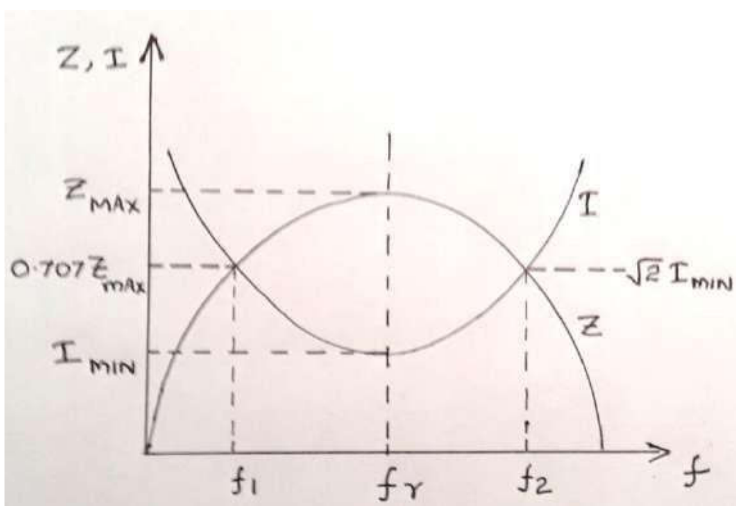


Fig: Variation of Impedance and Current with frequency in a Parallel RLC circuit

Here also the BW of the circuit is given by BW = f_2-f_1 where f_2 and f_1 are still called the upper and lower cut off frequencies but they are 3db higher cutoff frequencies since we notice that at these cutoff frequencies the amplitude of the current is √2 times higher than that of the amplitude of current at the resonant frequency.

The BW is computed here also on the same lines as we did for the series RLC circuit:

If the current at points P_1 and P_2 is √ 2 (3db) times higher than that of I_{min}(current at the resonant frequency) then the admittance of the circuit at points P_1 and P_2 is also √ 2 times higher than the admittance at f_r)

But amplitude of admittance at point P_1 is given by: Y = √ $1/R^2 + (1/\omega_1L - \omega_1C)^2$ and equating this to √ 2 /R we get

$1/\omega_1L - \omega_1C$ = 1/R ---------------(1)

Similarly amplitude of admittance at point P_2 is given by: Y = √ $1/R^2 + (\omega_2C - 1/\omega_2L)^2$ and equating this to √ 2 /R we get

$\omega_2C - 1/\omega_2L$ = 1/R --------------- (2)

Equating LHS of (1) and (2) and further simplifying we get

$$1/\omega_1L - \omega_1C = \omega_2C - 1/\omega_2L$$
$$1/\omega_1L + 1/\omega_2L = \omega_1C + \omega_2C$$
$$1/L\,[(\omega_1 + \omega_2)/\,\omega_1\omega_2] = (\omega_1 + \omega_2)C$$
$$1/L\,C = \omega_1\omega_2$$

Next adding the equations (1) and (2) above and further simplifying we get

$$1/\omega_1L - \omega_1C + \omega_2C - 1/\omega_2L = 2/R$$
$$(\omega_2C - \omega_1C) + (1/\omega_1L - 1/\omega_2L) = 2/R$$
$$(\omega_2 - \omega_1)C + 1/L\,[(\omega_2 - \omega_1)/\,\omega_1\omega_2] = 2/R$$

Substituting the value of $\omega_1\omega_2 = 1/LC$

$$(\omega_2 - \omega_1)C + LC/L\,[(\omega_2 - \omega_1)] = 2/R$$
$$(\omega_2 - \omega_1)C + C\,[(\omega_2 - \omega_1)] = 2/R$$
$$2\,C\,[(\omega_2 - \omega_1)] = 2/R \text{ Or } [(\omega_2 - \omega_1)] = 1/RC$$

From which we get the band width BW = f_2-f_1 = 1/2π RC

Dividing both sides by f_r we get : $(f_2-f_1)/\,f_r = 1/2\pi f_rRC$ -------- (1)

Quality factor of a Parallel RLC circuit:

The quality factor of a Parallel RLC circuit is defined as:

Q = Reactive power in Inductor (or Capacitor) at resonance / Average power at Resonance

Reactive power in Inductor at resonance = V^2/X_L

Reactive power in Capacitor at resonance = V^2/X_C

Average power at Resonance = V^2/R

Here the power is expressed in the form V^2/X (not as I^2X as in series circuit) since V is common across R,L and C in the parallel RLC circuit and it gets cancelled during the simplification.

Therefore Q = $(V^2/X_L) / (V^2/R) = (V^2/X_C) / (V^2/R)$

i.e. $Q = R/X_L = R/\omega_r L$ ....(1)

Or $Q = R/X_C = \omega_r RC$ ....(2)

From these two relations we can also define Q factor as:

Q = Resistance /Inductive (or Capacitive) reactance at resonance

Substituting the value of $\omega_r = 1/\sqrt{LC}$ in the expressions (1) or (2) for Q above we can get the value of Q in terms of R, L,C as below.

$$Q = (1/\sqrt{LC})RC = R(\sqrt{C/L})$$

Further using the relation $Q = \omega_r RC$ (equation 2 above) in the earlier equation (1) we got in BW viz. $(f_2-f_1)/f_r = 1/2\pi f_r RC$ we get : $(f_2-f_1)/f_r = 1/Q$ or $Q = f_r/(f_2-f_1) = f_r/BW$

i.e. In Parallel RLC circuit also the Q factor is inversely proportional to the BW.

Admittance, Conductance and Susceptance curves for a Parallel RLC circuit as a function of frequency:

- The effect of varying the frequency on the Admittance, Conductance and Susceptance of a parallel circuit is shown in the figure below.
- Inductive susceptance B_L is given by $B_L = -1/\omega L$. It is inversely proportional to the frequency ω and is shown in the in the fourth quadrant since it is negative.
- Capacitive susceptance B_C is given by $B_C = \omega C$. It is directly proportional to the frequency ω and is shown in the in the first quadrant as OP .It is positive and linear.
- Net susceptance $B = B_C - B_L$ and is represented by the curve JK. As can be seen it is zero at the resonant frequency f_r
- The conductance $G = 1/R$ and is constant
- The total admittance Y and the total current I are minimum at the resonant frequency as shown by the curve VW

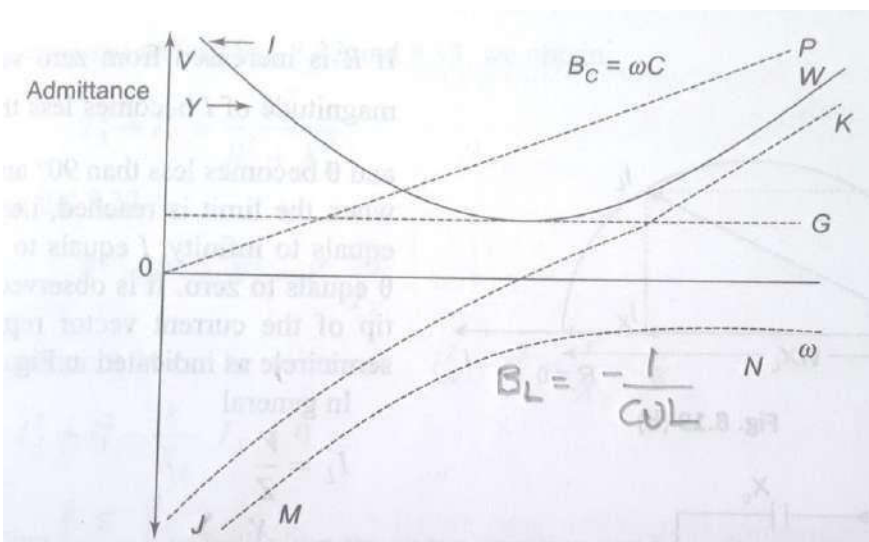


Fig: Conductance,Susceptance and Admittance plots of a Parallel RLC circuit

Current magnification in a Parallel RLC circuit:

Just as voltage magnification takes place across the capacitance and Inductance at the resonant frequency in a series RLC circuit , current magnification takes place in the currents through the capacitance and Inductance at the resonant frequency in a Parallel RLC circuit. This is shown below.

Voltage across the Resistance = V =IR

Current through the Inductance at resonance $I_L = V/ \omega_r L = IR / \omega_r L = I . R/ \omega_r L = I Q$ Similarly

Current through the Capacitance at resonance $I_C = V/ (1/\omega_r C) = IR / (1/\omega_r C) = I(R \omega_r C) = I Q$ From which we notice that the quality factor $Q = I_L / I$ or I_C / I and that the current through the inductance and the capacitance increases by Q times that of the current through the resistor at resonance. .

Important points In Parallel RLC circuit at resonant frequency:

- The impedance of the circuit becomes resistive and maximum i.e Z =R
- The current in the circuit becomes minimum
- The magnitudes of the capacitive Reactance and Inductive Reactance become equal
- The current through the Capacitor becomes equal and opposite to the current through the Inductor at resonance and is Q times higher than the current through the resistor

MODULE-IV

Time Response Analysis of (DC and AC) circuits

Introduction:

In this chapter we shall study transient response of the RL, RC series and RLC circuits with external DC excitations. Transients are generated in Electrical circuits due to abrupt changes in the operating conditions when energy storage elements like Inductors or capacitors are present. Transient response is the dynamic response during the initial phase before the steady state response is achieved when such abrupt changes are applied. To obtain the transient response of such circuits we have to solve the differential equations which are the governing equations representing the electrical behavior of the circuit. A circuit having a single energy storage element i.e. either a capacitor or an Inductor is called a Single order circuit and it's governing equation is called a First order Differential Equation. A circuit having both Inductor and a Capacitor is called a Second order Circuit and it's governing equation is called a Second order Differential Equation. The variables in these Differential Equations are currents and voltages in the circuit as a function of time.

A solution is said to be obtained to these equations when we have found an expression for the dependent variable that satisfies both the differential equation and the prescribed initial conditions. The solution of the differential equation represents the Response of the circuit. Now we will find out the response of the basic RL and RC circuits with DC Excitation.

RL CIRCUIT with external DC excitation:

Let us take a simple RL network subjected to external DC excitation as shown in the figure. The circuit consists of a battery whose voltage is *V* in series with a switch, a resistor *R*, and an inductor *L*. The switch is closed at $t = 0$.

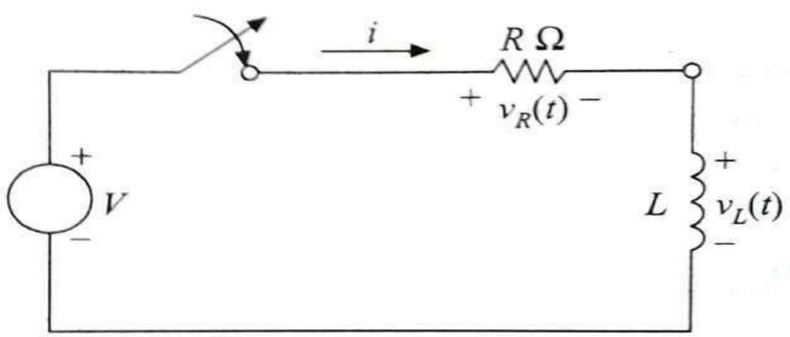


Fig: RL Circuit with external DC excitation

When the switch is closed current tries to change in the inductor and hence a voltage $V_L(t)$ is induced across the terminals of the Inductor in opposition to the applied voltage. The rate of change of current decreases with time which allows current to build up to it's maximum value.

It is evident that the current $i(t)$ is zero before $t = 0$.and we have to find out current $i(t)$for time t >0. We will find $i(t)$for time t >0 by writing the appropriate circuit equation and then solving it by separation of the variables and integration.

Applying Kirchhoff's voltage law to the above circuit we get :

$$V = v_R(t) + v_L(t)$$

$$i(t) = 0 \text{ for } t < 0 \text{ and}$$

Using the standard relationships of Voltage and Current for the Resistors and Inductors we can rewrite the above equations as

$$V = Ri + Ldi/dt \text{ for } t > 0$$

One direct method of solving such a differential equation consists of writing the equation in such a way that the variables are separated, and then integrating each side of the equation. The variables in the above equation are I and t. This equation is multiplied by dt and arranged with the variables separated as shown below:

$$Ri.\ dt + Ldi = V.\ dt$$

$$\text{i.e } Ldi = (V - Ri)dt$$

$$\text{i.e} \quad Ldi / (V - Ri) \quad = dt$$

Next each side is integrated directly to get:

$$-(L/R)\ \ln(V - Ri) = t + k$$

Where k is the integration constant. In order to evaluate k, an initial condition must be invoked. Prior to t = 0, i (t) is zero, and thus i (0−) = 0. Since the current in an inductor cannot change by a finite amount in zero time without being associated with an infinite voltage, we have i (0+) = 0. Setting i = 0 at t = 0, in the above equation we obtain

Thus, an expression for the response valid for all time t would be

$$i(t) = V/R\ [1 - e^{-Rt/L}]$$

This is normally written as:

$$i(t) = V/R\ [1 - e^{-t./\tau}]$$

where 'τ' is called the time constant of the circuit and it's unit is seconds.

The voltage across the resistance and the Inductorfor t >0can be written as :

$$vR(t) \quad = i(t).R \quad = V\ [1 - e^{-t./\tau}]$$

$$vL(t) = V - vR(t) = V - V\ [1 - e^{-t./\tau}] = V\ (e^{-t./\tau})$$

A plot of the currenti(t) and the voltages vR(t) & vL(t) is shown in the figure below.

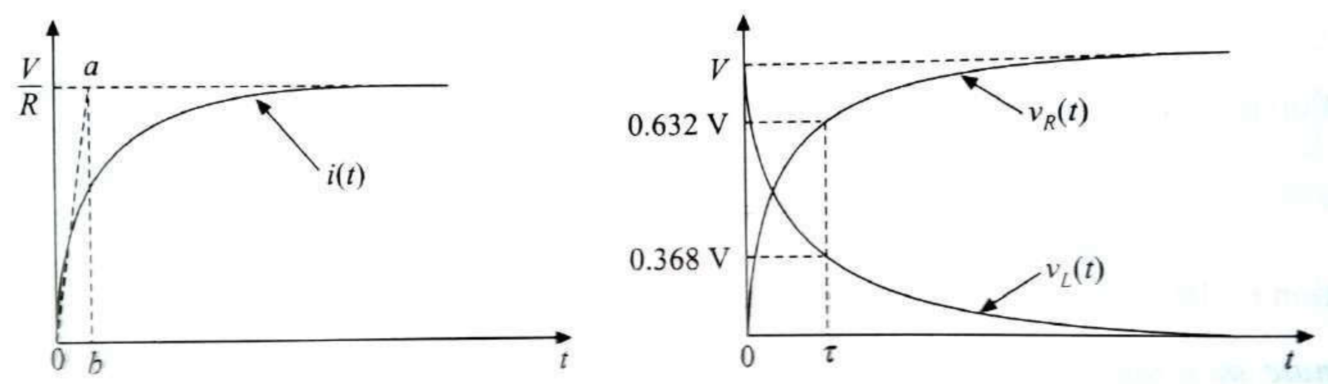


Fig: Transient current and voltages in the Series RL circuit.

At t = ‘τ’ the voltage across the inductor will be

$$v_L(\tau) = V\,(e^{-\tau/\tau}) = V/e = 0.36788\ V$$

And the voltage across the Resistor will be $v_R(\tau) = V\,[1 - e^{-\tau/\tau}] = 0.63212\ V$

The plots of current i(t) and the voltage across the Resistor $v_R(t)$ are called exponential growth curves and the voltage across the inductor $v_L(t)$ is called exponential decay curve.

RC CIRCUIT with external DC excitation:

A series RC circuit with external DC excitation V volts connected through a switch is shown in the figure below. If the capacitor is not charged initially i.e. it’s voltage is zero ,then after the switch S is closed at time t=0, the capacitor voltage builds up gradually and reaches it’s steady state value of V volts after a finite time. The charging current will be maximum initially (since initially capacitor voltage is zero and voltage across a capacitor cannot change instantaneously) and then it will gradually comedown as the capacitor voltage starts building up. The current and the voltage during such charging periods are called Transient Current and Transient Voltage.

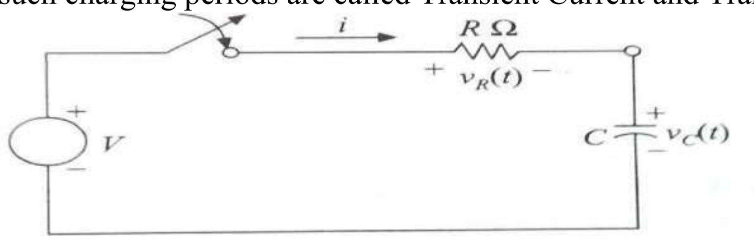


Fig: RC Circuit with external DC excitation

Applying KVL around the loop in the above circuit we can write

$$V = v_R(t) + v_C(t)$$

Using the standard relationships of voltage and current for an Ideal Capacitor we get

$$v_C(t) = (1/C)\int i(t)dt \quad \text{or} \quad i(t) = C.[dv_C(t)/dt]$$

and using this relation, $v_R(t)$ can be written as $v_R(t) = Ri(t) = R.\ C.[dv_C(t)/dt]$

Using the above two expressions for $v_R(t)$ and $v_C(t)$ the above expression for V can be rewritten as :

$$V = R.\ C.[dv_C(t)/dt] + v_C(t)$$

$$\text{Or finally } dv_C(t)/dt + (1/RC).\ v_C(t) = V/RC$$

The inverse coefficient of $v_C(t)$ is known as the time constant of the circuit τ and is given by $\tau = RC$ and it’s Units are seconds.

The above equation is a first order differential equation and can be solved by using the same method of separation of variables as we adopted for the LC circuit.

Multiplying the above equation $dv_C(t)/dt + (1/RC).\ v_C(t) = V/RC$

both sides by 'dt' and rearranging the terms so as to separate the variables $v_C(t)$ and t we get:

$$dv_C(t) + (1/RC).\, v_C(t) .\, dt = (V/RC).dt$$

$$dv_C(t) = [(V/RC)-(1/RC).\, v_C(t)].\, dt$$

$$dv_C(t) / [(V/RC)-(1/RC).\, v_C(t)] = dt$$

$$R.\, C .\, dv_C(t) / [(V-v_C(t)] = dt$$

Now integrating both sides w.r.t their variables i.e. '$v_C(t)$' on the LHS and 't' on the RHS we get

$$-RC \ln [V - v_C(t)] = t + k$$

where 'k' is the constant of integration. In order to evaluate k, an initial condition must be invoked. Prior to t = 0, $v_C(t)$ is zero, and thus $v_C(t)(0-) = 0$. Since the voltage across a capacitor cannot change by a finite amount in zero time, we have $v_C(t)(0+) = 0$. Setting $v_C(t) = 0$ at t = 0, in the above equation we obtain:

$$-RC \ln [V] = k$$

and substituting this value of $k = -RC \ln [V]$ in the above simplified equation $-RC \ln [V - v_C(t)] = t + k$ we get :

$$-RC \ln [V - v_C(t)] = t - RC \ln [V]$$

i.e. $-RC \ln [V - v_C(t)] + RC \ln [V] = t$ i.e. $-RC [\ln \{V - v_C(t)\} - \ln (V)] = t$

i.e. $[\ln \{V - v_C(t)\}] - \ln [V]\} = -t/RC$

i.e. $\ln [\{V - v_C(t)\}/(V)] = -t/RC$

Taking anti logarithm we get $[\{V - v_C(t)\}/(V)] = e^{-t/RC}$

i.e $v_C(t) = V(1 - e^{-t/RC})$

Which is the voltage across the capacitor as a function of time .

The voltage across the Resistor is given by : $v_R(t) = V - v_C(t) = V - V(1 - e^{-t/RC}) = V.e^{-t/RC}$

And the current through the circuit is given by: $i(t) = C.[dv_C(t)/dt] = (CV/CR)e^{-t/RC} = (V/R)e^{-t/RC}$

Or the other way: $i(t) = v_R(t)/R = (V.e^{-t/RC})/R = (V/R)e^{-t/RC}$

In terms of the time constant τ the expressions for $v_C(t)$, $v_R(t)$ and $i(t)$ are given by :

$$v_C(t) = V(1 - e^{-t/RC})$$

$$v_R(t) = V.e^{-t/RC}$$

$$i(t) = (V/R)e^{-t/RC}$$

The plots of current i(t) and the voltages across the resistor $v_R(t)$ and capacitor $v_C(t)$ are shown in the figure below.

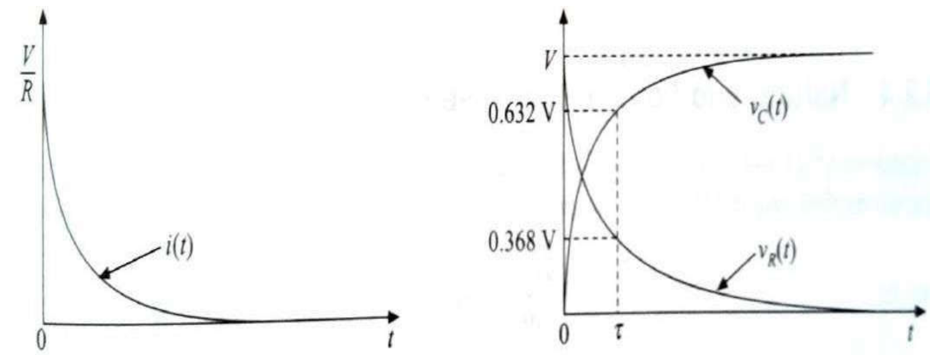


Fig : Transient current and voltages in RC circuit with DC excitation.

At t = 'τ' the voltage across the capacitor will be:

$$vC(\tau) = V\,[1 - e^{-\tau/\tau}\,] = 0.63212\,V$$

the voltage across the Resistor will be:

$$vR(\tau) = V\,(e^{-\tau\,/\tau}) = V/e = 0.36788\,V$$

and the current through the circuit will be:

$$i(\tau) = (V/R)\,(e^{-\tau\,/\tau}) = V/R.\,e = 0.36788\,(V/R)$$

Thus it can be seen that after one time constant the charging current has decayed to approximately 36.8 % of it's value at t=0 . At t= 5 τ charging current will be

$$i(5\tau) = (V/R)\,(e^{-5\tau\,/\tau}) = V/R.\,e^{5} = 0.0067(V/R)$$

This value is very small compared to the maximum value of (V/R) at t=0 .Thus it can be assumed that the capacitor is fully charged after 5 time constants.

The following similarities may be noted between the equations for the transients in the LC and RC circuits:

- The transient voltage across the Inductor in a LC circuit and the transient current in the RC circuit have the same form $k.(e^{-t\,/\tau})$
- The transient current in a LC circuit and the transient voltage across the capacitor in the RC circuit have the same form $k.(1-e^{-t\,/\tau})$

But the main difference between the RC and RL circuits is the effect of resistance on the duration of the transients.

- In a RL circuit a large resistance shortens the transient since the time constant τ =L/R Becomes small.
- Where as in a RC circuit a large resistance prolongs the transient since the time constant τ = RC becomes large.

Discharge transients: Consider the circuit shown in the figure below where the switch allows both charging and discharging the capacitor. When the switch is position 1 the capacitor gets charged to the applied voltage V. When the switch is brought to position 2, the current discharges from the positive terminal of the capacitor to the negative terminal through the resistor R as shown in the figure (b). The circuit in position 2 is also called source free circuit since there is no any applied voltage.

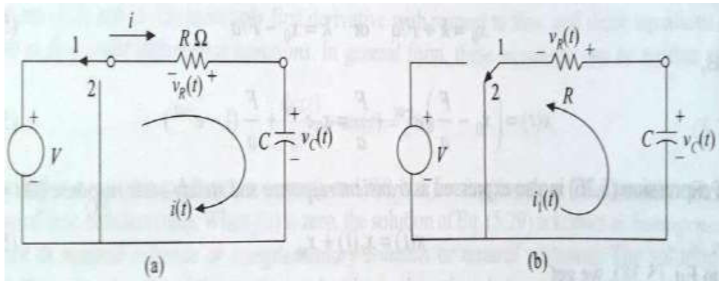


Fig: RC circuit (a) During Charging (b) During Discharging

The current i_1 flow is in opposite direction as compared to the flow of the original charging current i. This process is called the discharging of the capacitor. The decaying voltage and the current are called the discharge transients. The resistor, during the discharge will oppose the flow of current with the polarity of voltage as shown. Since there is no any external voltage source, the algebraic sum of the voltages across the Resistance and the capacitor will be zero (applying KVL) .The resulting loop equation during the discharge can be written as

$$v_R(t)+v_C(t) = 0 \text{ or } v_R(t) = -v_C(t)$$

We know that $v_R(t) = R.i(t) = R.C.dv_C(t)/dt$. Substituting this in the first loop equation we get

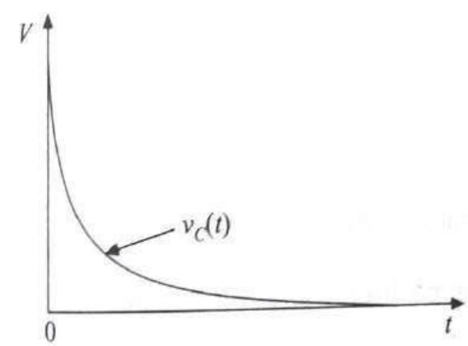
$$R.C.dv_C(t)/dt + v_C(t) = 0$$

The solution for this equation is given by $v_C(t) = Ke^{-t/\tau}$ where K is a constant decided by the initial conditions and $\tau = RC$ is the time constant of the RC circuit

The value of K is found out by invoking the initial condition $v_C(t) = V$ @t = 0

Then we get K = V and hence $v_C(t) = Ve^{-t/\tau}$; $v_R(t) = -Ve^{-t/\tau}$ and $i(t) = v_R(t)/R = (-V/R)e^{-t/\tau}$

The plots of the voltages across the Resistor and the Capacitor are shown in the figure below



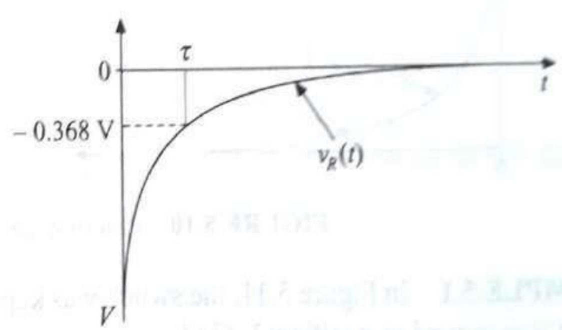


Fig: Plot of Discharge transients in RC circuit

Decay transients: Consider the circuit shown in the figure below where the switch allows both growing and decaying of current through the Inductance. When the switch is position 1 the current through the Inductance builds up to the steady state value of V/R. When the switch is brought to position 2, the current decays gradually from V/R to zero. The circuit in position 2 is also called a source free circuit since there is no any applied voltage.

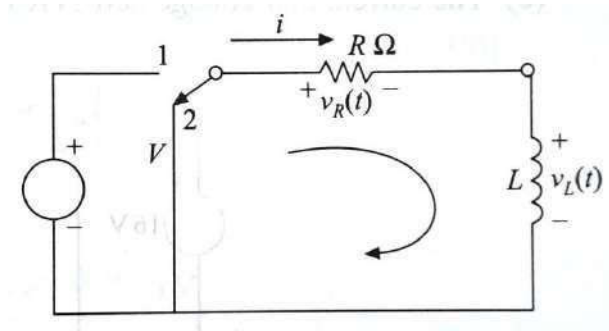


Fig: Decay Transient In RL circuit

The current flow during decay is in the same direction as compared to the flow of the original growing /build up current. The decaying voltage across the Resistor and the current are called the *decay transients*.. Since there is no any external voltage source , the algebraic sum of the voltages across the Resistance and the Inductor will be zero (applying KVL) .The resulting loop equation during the discharge can be written as

$$vR(t)+vL(t) = R.i(t) + L.di(t)/dt = 0 \qquad \text{and } vR(t) = - vL(t)$$

The solution for this equation is given by $i(t) = Ke^{-t/\tau}$ where K is a constant decided by the initial conditions and τ =L/R is the time constant of the RL circuit.

The plts of the voltages across the Resistor and the Inductor and the decaying current through the circuit are shown in the figure below.

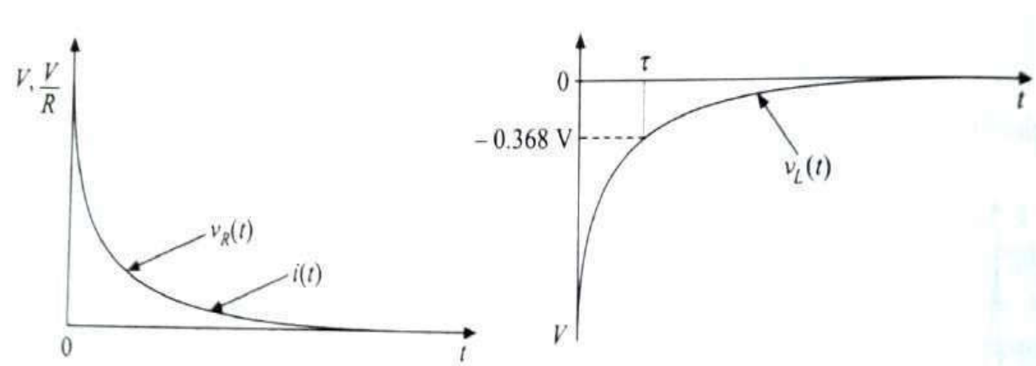


Fig: Plot of Decay transients in RL circuit

The Concept of Natural Response and forced response:

The RL and RC circuits we have studied are with external DC excitation. These circuits without the external DC excitation are called source free circuits and their Response obtained by solving the corresponding differential equations is known by many names. Since this response depends on the general nature of the circuit (type of elements, their size, their interconnection method etc.,) it is often called a Natural response. However any real circuit we construct cannot store energy forever. The resistances intrinsically associated with Inductances and Capacitors will eventually dissipate the stored energy into heat. The response eventually dies down,. Hence it is also called Transient response. As per the mathematician's nomenclature the solution of such a homogeneous linear differential equation is called Complementary function.

When we consider independent sources acting on a circuit, part of the response will resemble the nature of the particular source. (Or forcing function) This part of the response is called particular solution. , the steady state response or forced response. This will be complemented by the complementary function produced in the source free circuit. The complete response of the circuit is given by the sum of the complementary function and the particular solution. In other words:

The Complete response = Natural response + Forced response

There is also an excellent mathematical reason for considering the complete response to be composed of two parts—the forced response and the natural response. The reason is based on the fact that the solution of any linear differential equation may be expressed as the sum of two parts: the complementary solution (natural response) and the particular solution (forced response).

Determination of the Complete Response:

Let us use the same RL series circuit with external DC excitation to illustrate how to determine the complete response by the addition of the natural and forced responses. The circuit shown in the figure

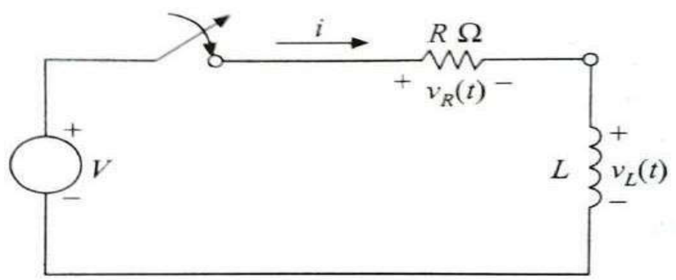


Fig: RL circuit with external DC excitation

was analyzed earlier, but by a different method. The desired response is the current *i (t)*, and now we first express this current as the sum of the natural and the forced current,

$$i = i_n + i_f$$

The functional form of the natural response must be the same as that obtainedwithout any sources. We therefore replace the step-voltage source by a short circuit and call it the *RL source free* series loop. And i_n can be shown to be :

$$i_n = Ae^{-Rt/L}$$

where the amplitude *A*is yet to be determined; since the initial conditionapplies to the*complete* response, we cannot simply assume $A = i(0)$.We next consider the forced response. In this particular problem theforced response is constant, because the source is a constant *V*for allpositive values of time. After the natural response has died out, there can beno voltage across the inductor; hence the all ythe applied voltage *V* appears across *R*, and theforced response is simply

$$i_f = V/R$$

Note that the forced response is determined completely. There is no unknown amplitude. We next **combine the two responses to obtain:**

$$i = Ae^{-Rt/L} + V/R$$

And now we have to apply the initial condition to evaluate *A*. The current is zero prior to *t* = 0, and it cannot change value instantaneously since it is the current flowing through an inductor. Thus, the current is zero immediately after *t* = 0, and

$$A + V/R = 0$$

So that

$$A = -V/R$$

And $i = (V/R)(1 - e^{-Rt/L})$

Note carefully that *A* is not the initial value of i, since A = −V/R, while i (0) = 0.

But In source-free circuits, A would be the initial value of the response given by $i_n = I_0 e^{-Rt/L}$ (where I_0 =A is the current at time t=0). When forcing functions are present, however,we must first find the initial value of the complete response and then substitute this in the equation for the complete response to find A.Then this value of A is substituted in the expression for the total response i

Amoregeneral solution approach:

The method of solving the differential equation by separating the variables or by evaluating the complete response as explained above may not be possible always. In such cases we will rely on a verypowerful method, the success of which will depend upon our intuition or experience. We simply guess or assume a form for the solution and then test our assumptions, first by substitution in the differential equation, and then by applying the given initial conditions. Since we cannot be expected to guess the exact numerical expression for the solution, we will assume a solution containing several unknown constants and select the values for these constants in order to satisfy the differential equation and the initial conditions.

In order to satisfy this equation for all values of time, it is necessary thatA = 0, or $s_1 = -\infty$, or $s_1 = -R/L$. But if A = 0 or $s_1 = -\infty$, then everyresponse is zero; neither can be a solution to our problem. Therefore, wemust choose

$$s_1 = -R/L$$

And our assumed solution takes on the form:

$$i(t) = A.e^{-Rt/L}$$

The remaining constant must be evaluated by applying the initial conditioni (0) = I_0. Thus,A = I_0, and the final form of the assumed solution is(again):

$$i(t) = I_0.e^{-Rt/L}$$

A Direct Route: The Characteristic Equation:

In fact, there is a more direct route that we can take. To obtain the solution for the first order DEwe solveds1 + R/L= 0 which is known as the *characteristic equation*and then substituting this value of s_1=- R/Lin the assumed solutioni (t) = $A.e^{s_1 t}$ which is same in this direct method also. We can obtain the characteristic equation directly from the differential equation, without the need for substitution of our trial solution. Consider the general first-order differential equation:

$$a(df/dt) + bf = 0$$

Where a and b are constants. We substitute s for the differentiation operator d/dt in the original differential equation resulting in

$$a(d f/dt) + bf = (as + b) f = 0$$

From this we may directly obtain the characteristic equation: $as + b = 0$
which has the single root $s = -b/a$. Hence the solution to our differential equationis then given by :

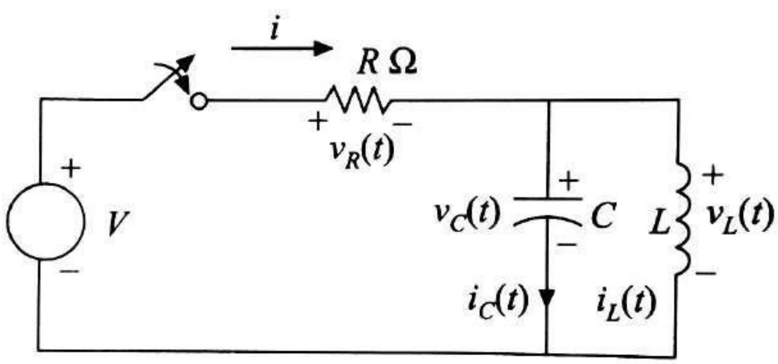
$$f = A.e^{-bt/a}$$

This basic procedure can be easily extended to second-order differential equations which we will encounter for RLC circuits and we will find it useful since adopting the variable separation method is quite complex for solving second order differential equations.

RLC CIRCUITS:

Earlier, we studied circuits which contained only one energy storage element, combined with a passive network which partly determined how long it took either the capacitor or the inductor to charge/discharge. The differential equations which resulted from analysis were always first-order. In this chapter, we consider more complex circuits which contain both an inductor and acapacitor. The result is a second-order differential equation for any voltage or current of interest. What we learned earlier is easily extended to the study of these so-called *RLC* circuits, although now we need two initial conditions to solve each differential equation. There are two types of RLC circuits: *Parallel RLC circuits* and *Series circuits* .Such circuits occur routinely in a wide variety of applications and are very important and hence we will study both these circuits.

Parallel RLC circuit:



Let us first consider the simple parallel RLC circuit with DC excitation as shown in the figure below.

Fig: Parallel *RLC* circuit with DC excitation.

For the sake of simplifying the process of finding the response we shall also assume that the initial current in the inductor and the voltage across the capacitor are zero. Then applying the Kirchhoff's current law (KCL)($i = i_C + i_L$)to the common node we get the following integro differential equation:

$$(V-v)/R = 1/L\int_{-\infty}^{t} v\,dt' + C.dv/dt$$

$$V/R = v/R + 1/L\int_{-\infty}^{t} v\,dt' + C.dv/dt$$

Where $v = v_C(t) = v_L(t)$ is the variable whose value is to be obtained .

When we differentiate both sides of the above equation once with respect to time we get thestandard Linear second-order homogeneous differential equation

$C.(d^2v/dt^2)+ (1/R).(dv/dt)+ (1/L).v =$

$0\ (d^2v/dt^2)+ (1/RC).(dv/dt)+ (1/LC).v$

$= 0$

whose solution v(t) is the desired

response. This can be written in the

form:

$$[s^2 + (1/RC)s + (1/LC)].v(t) = 0$$

where 's' is an operator equivalent to (d/dt) and the corresponding *characteristic equation*(as explained earlier as a direct route to obtain the solution) is then given by :

$$[s^2 + (1/RC)s + (1/LC)] = 0$$

This equation is usually called the auxiliary equation or the characteristic equation, as we discussed earlier .If it can be satisfied, then our assumed solution is correct. This is a quadratic equation and the roots s1 and s2are given as:

$$s1= - 1/2RC+\sqrt{[(1/2RC)^2- 1/LC]}\quad s2= - 1/2RC-\sqrt{[(1/2RC)^2- 1/LC\,]}$$

And we have the general form of the response as :

$$v(t) = A1e^{s1t}+ A2e^{s2t}$$

where $s1$ and $s2$ are given by the above equations and $A1$ and $A2$ are two arbitrary constants which are to be selected to satisfy the two specified initial condition

Definition of Frequency Terms:

The form of the natural response as given above gives very little insight in to the nature of the curve we might obtain if *v(t)*were plotted as a function of time. The relative amplitudes of $A1$ and $A2$, for example, will certainly be important in determining the shape of the response curve. Further the constants $s1$ and $s2$ can be real numbers or conjugate complex numbers, depending upon the values of R, L, and Cin the given network. These two cases will produce fundamentally different response forms. Therefore, it will be helpful to make some simplifying substitutions in the equations for $s1$ and $s2$.Since the exponents $s1t$and $s2t$ must be dimensionless, $s1$ and $s2$ must have the unit of some dimensionless quantity "per second." Hence in the equations for $s1$ and $s2$ we see that the units of $1/2RC$ and $1/\sqrt{LC}$ must also be s^{-1}(i.e., seconds^{-1}). Units of this type are called frequencies.

Now two new terms are defined as below:

$$\omega 0 = 1/\sqrt{LC}$$

$$\alpha = 1/2RC$$

Which is termed as resonant frequency
and
which is termed as the exponential damping coefficient

α the exponential damping coefficient is a measure of how rapidly the natural response decays or damps out to its steady, final value(usually zero). And s, s1, and s2, are called complex frequencies.
We should note that s1, s2, α, and ω0 are merely symbols used to simplify the discussion of RLC circuits. They are not mysterious new parameters of any kind. It is easier, for example, to say "alpha" than it is to say "the reciprocal of 2RC."

Now we can summarize these results.
The response of the parallel RLC circuit is given by :

$$v(t) = A1e^{s1t} + A2e^{s2t} \cdots\cdots [1]$$

where

$$s1 = -\alpha + \sqrt{\alpha^2 - \omega 0^2} \cdots\cdots [2]$$

$$s2 = -\alpha - \sqrt{\alpha^2 - \omega 0^2} \cdots\cdots [3]$$

$$\alpha = 1/2RC \ldots\ldots\ldots\ldots\ldots [4]$$

and

$$\omega 0 = 1/\sqrt{LC} \ldots\ldots [5]$$

*A*1 and *A*2must be found by applying the given initial conditions.
We note three basic scenarios possible with the equations for s1 and s2 depending on the relative values of α and ω0 (which are in turn dictated by the values of R, L, and C).
Case A:
$\alpha > \omega 0$,i.e when $(1/2RC)^2 > 1/LC$s1 and s2 will both be negative real numbers, leading to what is referred to as an over damped response given by :

$$v(t) = A1e^{s1t} + A2e^{s2t}$$

Sinces1 and s2are both negative real numbers this is the (algebraic) sum of two decreasing exponential terms. Sinces2 is a larger negative number it decays faster and then the response is dictated by the first term $A1e^{s1t}$.
Case B :
$\alpha = \omega 0$, ,i.e when $(1/2RC)^2 = 1/LC$, s1 and s2are equal which leads to what is called a critically damped response given by :

$$v(t) = e^{-\alpha t}(A1t + A2)$$

Case C :
$\alpha < \omega 0$,i.e when $(1/2RC)^2 < 1/LC$ both s1 and s2 will have nonzero imaginary components, leading to what is known as an under damped response given by :

$$v(t) = e^{-\alpha t}(A1 \cos\omega d\, t + A2 \sin\omega d\, t)$$

where ωdis called *natural resonant frequency* and is given given by:

$$\omega d = \sqrt{\omega 0^2 - \alpha^2}$$

We should also note that the general response given by the above equations [1] through [5] describe not only the voltage but all three branch currents in the parallel *RLC* circuit; the constants $A1$ and $A2$ will be different for each, of course.

Transient response of a series RLC circuit:

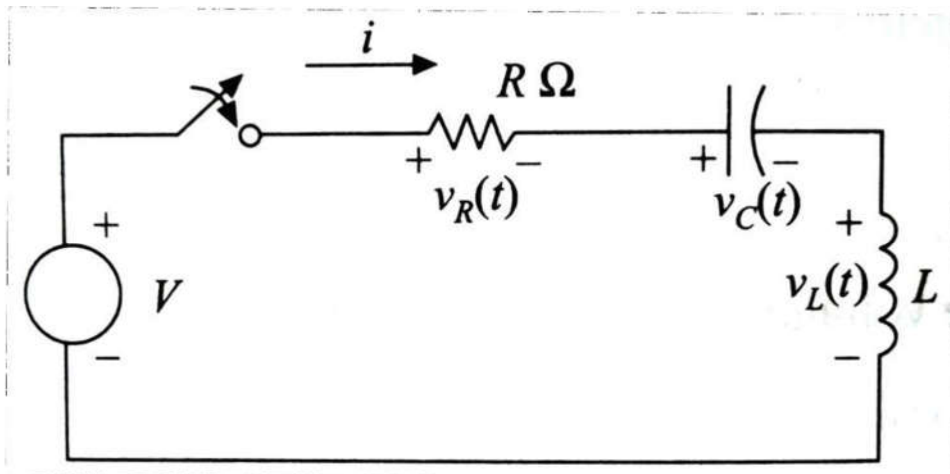


Fig: Series RLC circuit with external DC Excitation

Applying KVL to the series RLC circuit shown in the figure above at t= 0 gives the following basic relation :

$$V = vR(t) + vC(t) + vL(t)$$

Representing the above voltages in terms of the current iin the circuit we get the following differential equation:

$$Ri + 1/C\int \square\square\square + L.\,(di/dt)= V$$

To convert it into a differential equation it is differentiated on both sides with respect to time and we get

$$L(d^2i/dt^2)+ R(di/dt)+ (1/C)i = 0$$

This can be written in the form

$[S^2 + (R/L)s + (1/LC)].i = 0$ where ‘s’ is an operator equivalent to (d/dt)

And the corresponding characteristic equation is then given by

$$[s^2 + (R/L)s + (1/LC)] = 0$$

This is in the standard quadratic equation form and the roots$s1$and$s2$are given by

$$s1,s2 =- R/2L\pm\sqrt{[(R/2L)^2- (1/LC)]} = -\alpha \pm\sqrt{(\alpha^2- \omega0^2)}$$

Where α is known as the same exponential damping coefficient and $\omega0$is known as the same Resonant frequency as explained in the case of Parallel RLC circuit and are given by :

$$\alpha = R/2L \text{ and } \omega0= 1/ \sqrt{LC}$$

and $A1$ and $A2$must be found by applying the given initial conditions.

Here also we note three basic scenarios with the equations for s1 and s2 depending on the relative sizes of α and $\omega 0$ (dictated by the values of R, L, and C).

Case A:
$\alpha > \omega 0$,i.e when $(R/2L)^2 > 1/LC$, s1 and s2 will both be negative real numbers, leading to what is referred to as an over damped response given by :

$$i(t) = A1e^{s1t} + A2e^{s2t}$$

Sinces1 and s2are both be negative real numbers this is the (algebraic) sum of two decreasing exponential terms. Sinc s2 is a larger negative number it decays faster and then the response is dictated by the first term $A1e^{s1t}$.

Case B :
$\alpha = \omega 0$, ,i.e when $(R/2L)^2 = 1/LC$s1 and s2are equal which leads to what is called a critically damped response given by :

$$i(t) = e^{-\alpha t}(A1t + A2)$$

Case C :
$\alpha < \omega 0$,i.e when $(R/2L)^2 < 1/LC$both s1 and s2 will have nonzero imaginary components, leading to what is known as an under damped responsegiven by :

$$i(t) = e^{-\alpha t}(A1 \cos\omega d\, t + A2 \sin\omega d\, t)$$

where ωd is called natural resonant frequency and is given given by:

$$\omega d = \sqrt{\omega 0^2 - \alpha^2}$$

Here the constants A1 and A2 have to be calculated out based on the initial conditions case by case
Summary of the Solution Process:
In summary, then, whenever we wish to determine the transient behavior of a simple three-element RLC circuit, we must first decide whether it is a series or a parallel circuit, so that we may use the correct relationship for α. The two equations are
$\alpha = 1/2RC$ (parallel RLC)
$\alpha = R/2L$ (series RLC)
Our second decision is made after comparing α with $\omega 0$, which is given for either circuit by
$\omega 0 = 1/\sqrt{LC}$

- If$\alpha > \omega 0$, the circuit is over damped, and the natural response has the for

Where $fn(t) = A1e^{s1t} + A2e^{s2t}$ $s1, 2 = -\alpha \pm\sqrt{(\alpha^2 - \omega 0^2)}$

- If $\alpha = \omega 0$, then the circuit is critically damped and

$$fn(t) = e^{-\alpha t}(A1t + A2)$$

- And finally, if$\alpha < \omega 0$, then we are faced with the underdamped response, where

$$f_n(t) = e^{-\alpha t}(A_1 \cos\omega_d t + A_2 \sin\omega_d t)$$

$$\omega_d = \sqrt{(\omega_0^2 - \alpha^2)}$$

Solution using Laplace transformation method: In this topic we will study Laplace transformation method of finding solution for the differential equations that govern the circuit behavior. This method involves three steps:

- First the given Differential equation is converted into "s" domain by taking it's Laplace transform and an algebraic expression is obtained for the desired variable
- The transformed equation is split into separate terms by using the method of Partial fraction expansion
- Inverse Laplace transform is taken for all the individual terms using the standard inverse transforms.

The expression we get for the variable in time domain is the required solution.

For the ease of reference a table of important transform pairs we use frequently is given below.

Table of Important Transform pairs

$f(t)$ (*Function*)	$F(s)$ (*Laplace Transform*)
$u(t)$ (unit step)	$1/s$
$\delta(t)$ (unit impulse)	1
e^{-at}	$\frac{1}{(s+a)}$
$\sin \omega t$	$\frac{w}{(s^2+\omega^2)}$
$\cos \omega t$	$\frac{s}{(s^2+\omega^2)}$
$e^{-at} \sin \omega t$	$\frac{\omega}{(s+a)^2+\omega^2}$
$e^{-at} \cos \omega t$	$\frac{(s+a)}{(s+a)^2+\omega^2}$
t	$1/s^2$
$\frac{df(t)}{dt}$	$sF(s)$
$\int f(t)dt$	$F(s)/s$

This method is relatively simpler compared to Solving the Differential equations especially for higher order differential equations since we need to handle only algebraic equations in 's' domain.

This method is illustrated below for the series RL,RC and RLC circuits.

Series RL circuit with DC excitation:

Let us take the *series RL* circuit with external DC excitation shown in the figure below.

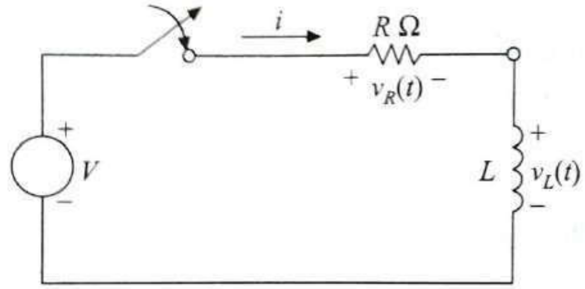


Fig: RL Circuit with external DC excitation

The governing equation is same as what we obtained earlier.

$$V = Ri + L\,di/dt \text{ for } t > 0$$

Taking inverse transform of the above expression for I(s)using the standard transform pairs we get the solution for i(t) as

$$i(t) = (V/R) - (V/R).e^{-(R/L)t} = (V/R)(1 - e^{-(R/L)t})$$

Which is the same as what we got earlier by solving the governing differential equation directly.

RC Circuit with external DC excitation:

Let us now take the *series RC* circuit with external DC excitation shown in the figure below.

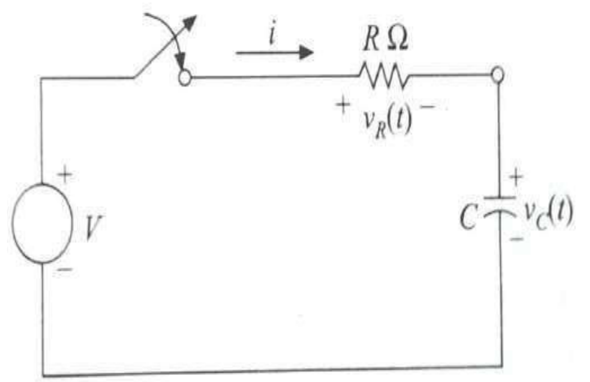


Fig: RC Circuit with external DC excitation

The governing equation is same as what we obtained earlier and is worked out again for easy understanding :

Now we will take Laplace transform of the above equation using the standard Transform pairs and rules:

$$V/s = R.C.s.vC(s) + vC(s)$$

$$V/s = vC(s)\,(R.C.s. + 1)$$

$$vC(s) = (V/s)/(R.C.s + 1)$$

$$vC(s) = (V/RC)/[s.(s + 1/RC)]$$

Now expanding this equation into partial fractions we get

$$vC(s) = (V/RC)/[s.(s + 1/RC)] = A/s + B/(s + 1/RC) \quad (1)$$

Where $A = (V/RC)/(1/RC)] = V$ and $B = (V/RC)/-(1/RC)] = -V$
Substituting these values of A and B into the above equation (1) for $v_C(s)$ we get

$$v_C(s) = (V/s) - [V/(s + 1/RC)] = V[(1/s) - \{1/(s + 1/RC)\}]$$

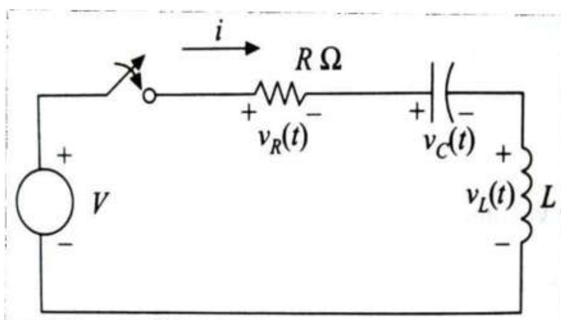
And now taking the inverse Laplace transform of the above equation we get

$$v_C(t) = V(1 - e^{-t/RC})$$

which is the voltage across the capacitor as a function of time and is the same as what we obtained earlier by directly solving the differential equation.
And the voltage across the Resistor is given by $v_R(t) = V - v_C(t) = V - V(1 - e^{-t/RC}) = V.e^{-}$

And the current through the circuit is given by $i(t) = C.[dv_C(t)/dt] = (CV/RC)e^{-t/RC} = (V/R)e^{-t/RC}$

Series RLC circuit with DC excitation:



RL circuit with external DC excitation (Charging Transient) :

- $i(t) = V/R\,[1 - e^{-t./\tau}]$
- $v_L(t) = V\,(e^{-t./\tau})$
- $v_R(t) = i(t).R = V\,[1 - e^{-t./\tau}]$

Source free RL circuit (Decay Transients) :

- $i(t) = (V/R)\,.\,e^{-t/\tau}$; $v_R(t) = R.i(t) = Ve^{-t/\tau}$ and $v_L(t) = -Ve^{-t/\tau}$

RC circuit with external DC excitation (Discharge Transients):

- $v_C(t) = V(1 - e^{-t/RC})$
- $v_R(t) = V.\,e^{-t/RC}$
- $i(t) = (V/R)\,e^{-t/RC}$

Source free RC circuit (Discharge transients) :

- $vC(t) = Ve^{-t/\tau}$; $vR(t) = -Ve^{-t/\tau}$ and $i(t) = vR(t)/R = (-V/R)e^{-t/\tau}$

Series RLC circuit: For this circuit three solutions are possible:

1. $\alpha > \omega0$, i.e when $(R/2L)^2 > 1/LC$, s1 and s2 will both be negative real numbers, leading to what is referred to as an over damped responsegiven by:

$$i(t) = A1e^{s1t} + A2e^{s2t}$$

2. $\alpha = \omega0$, , i.e when $(R/2L)^2 = 1/LC$ s1 and s2 are equal which leads to what is called a critically damped response given by:

$$i(t) = e^{-\alpha t}(A1t + A2)$$

3. $\alpha < \omega0$, i.e when $(R/2L)^2 < 1/LC$ both s1 and s2 will have nonzero imaginary components, leading to what is known as an under damped response given by :

   1. Voltage across Inductor $vL = V - vR$
      But it is easier to find using the second method. ∴ $vL = 100 -- 100(1—e^{-5t})$

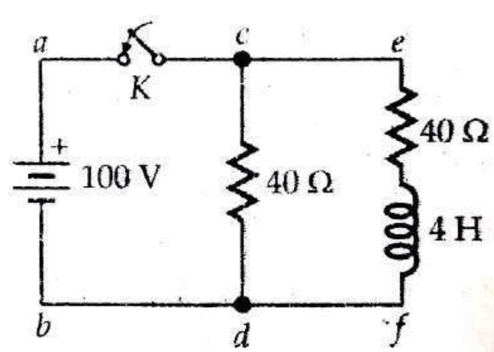
$$vL = 100.\ e^{-5t}$$

(b) At time t= 0.5 secsi(t) = $4(1–e^{5t}) = 4(1 - e^{-2.5}) = 3.67$ Amps
(c) To find out the time at which the voltages across the Inductor and the Resistor are equal we can equate the expressions for $vR = 100(1—e^{-5t})$ and $vL = 100.e^{-5t}$ and solve for t. But the simpler method is, we know that since the applied voltage is 100 V the condition vR =vLwill also be satisfiedwhenvR =vL = 50 V. i.e $vR = 100(1—e^{-5t})$ = 50 volts and $vL = 100.e^{-5t} = 50$ V. We will solve the second equation [$vL = 100.\ e^{-5t} = 50$ V] to get t which is easier.

$e^{-5t} = 50/100 = 0.5$.
Taking natural logarithm on both sides we get:
--5t .ln(e) = ln 0.5 i.e --5t .1 = -0.693 i.e t = 0.693/5 = 0.139 secs

∴ The voltages across the resistance and the Inductance are equal at time t = 0.139 secs

Example 3: In the figure shown below after the steady state condition is reached , at time t=0 the switch K is suddenly opened. Find the value of the current through the inductor at time t = 0.5 seconds.



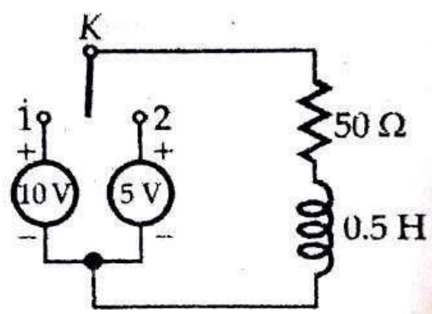
Solution: The current in the path *acdb* (through the resistance of 40 Ω alone) is 100/40 = 2.5Amps.(Both steady state and transient are same)
The steady state current through the path *aefb* (through the resistance of 40 Ω and inductance of 4H) is also = 100/40 = 2.5 Amps.

Now when the switch K is suddenly opened, the current through the path *acdb*(through the resistance of 40 Ω alone) immediately becomes zero because this path contains only resistance. But the current through the inductor decays gradually but now through the different path *efdce*
The decay current through a closed RL circuit is given by $I.e^{-t/\tau}$ where I is the earlier steady state current of 2.5 amps through L and τ = L/R of the decay circuit. *It is to be noted carefully here that in the decay path both resistors are there and hence R =40+40 = 80Ω*
Hence τ= L/R = 4/80 = 0.05 secs

Hence the current through the inductor at time 0.5 secs is given by i(t) @0.5secs $=2.5.e^{-0.5/0.05}$
i.e i(t) @0.5secs $= 2.5.e^{-10}$

i.e i(t) @0.5secs= 1.14×10^{-4} Amps

Example 4: In the circuit shown below the switch is closed to position 1 at time t = 0 secs. Then at time t = 0.5 secs the switch is moved to position 2. Find the expressions for the current through the circuit from 0 to 0. 5 msecs and beyond 0. 5 msecs.
Solution:The time constant $\tau$of the circuit in both the conditions is same and is given by τ = L/R = 0.5/50 = 0.01 secs



1. During the time t=0 to 0.5 msecs. i(t) is given by the standard expression for growing current through a L R circuit: i(t)during 0 to 0.5 msecs = V/R ($1-e^{-t/\tau}$)

i(t)during 0 to 0.5 msecs = V/R ($1-e^{-t/0.01}$) Amps

And the current i(t) @ t= 0.5 msecs = 10/50 (1-- $e^{-0.5\times10^{-3}/0.01}$) = 0.2 (1 – $e^{-0.05}$) = 9.75 mA

i(t) @ t = 0.5 msecs = 9.75 mA and this would be the initial current when the switch is moved to position 2

2. During the time beyond 0.5 msecs (switch is in position 2): The initial current is 9.75 mA . The standard expression for the growing current*i(t) = V/R ($1-e^{-t/\tau}$)* is not applicable now since it has been derived with initial condition of i(t) =0 at t=0 where as the initial condition for the current i(t) now in position 2 is 9.75 mA . Now an expression for i(t) in position 2 is to be derived from first principles taking fresh t=0 and initial current i(0) as 9.75mA.
The governing equation in position 2 is given by :

$$50i+0.5di/dt = 5$$

We will use the same *separation of variables method* to solve this differential equation. Dividing the above equation by 0.5, then multiplying by dt and separating the terms containing the two variables i and t we get:
100i + di/dt = 10 i.e 100i.dt +di = 10.dt i.e di = dt (10 – 100i) i.e di/ (10 – 100i) = dt
Now integrating on both sides we get

--1/100 ln (10 -- 100i) = t + K ------------- (1)The constant K is now to be evaluated by invoking the new initial condition i(t) = 9.75 mAat t =0

$--1/100 \ln (10 – 100\text{x}9.75\text{X}10^{--3})= K = --1/100 \ln (10 – 0.975) = --1/100 \ln (9.025)$

Substituting this value of K in the above equation (1) we get

--1/100 ln (10 -- 100i) = t --1/100 ln (9.025)
--1/100 ln (10 -- 100i) + 1/100 ln (9.025) = t
--1/100 [ln (10 -- 100i) -- ln (9.025)] = t
--1/100 . ln [(10 -- 100i) / (9.025)] = t
ln [(10 -- 100i) / (9.025)] = --100t

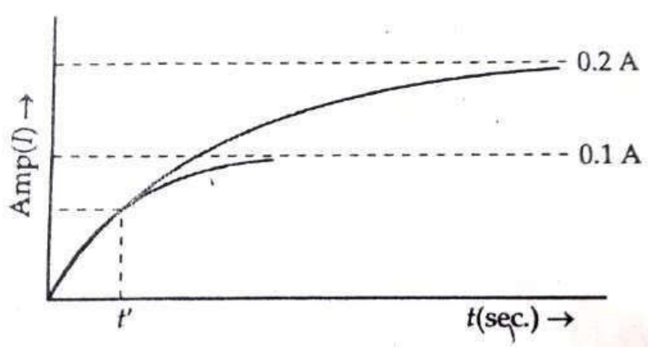
Taking antilogarithm to base e on both sides we get:

$(10 -- 100i) / (9.025)] = e^{--100t}$ $(10 -- 100i) = 9.025 \text{ x } e^{--100t}$ $(10 --9.025 \text{ x } e^{--100t}) = 100i$

$i = (10 --9.025 \text{ x } e^{--100t})/ 100 = 10/100 -- 9.025 \text{ x } e^{-100t} /100$

And finally $i = 0.1 –0.09. e^{--100t}$

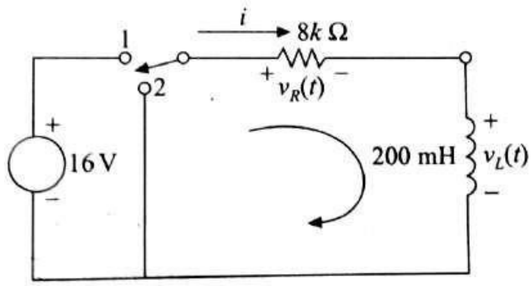
The currents during the periods t = o to 0.5 mses and beyond t = 0.5msec are shown in the figure below. Had the switch been in position 1 all through, the current would have reached the steady state value of 0.2 amps corresponding to source voltage of 10 volts as shown in the top curve. But since the switchis changed to position 2 the current changed it's path towards the new steady state current of 0.1 Amps corresponding the new source voltage of 5 Volts from 0.5 msecs onwards.



Example 5: In the circuit shown below the switch is kept in position 1 upto 250 µsecs and then moved to position 2. Find

(a) The current and voltage across the resistor at t = 100 μsecs
(b) The current and voltage across the resistor at t = 350 μsecs

Solution : The time constant τ of the circuit is given by $\tau = L/R = 200\text{mH}/8\text{K}\Omega = 25$ μsec and is same in both the switch positions.



(a)The current in the circuit upto 250 μsec (till switch is in position 1) is given by : $i(t)$ growing $= V/R\,(1 - e^{-t/\tau}) = (16/8)X10^{--3}\,(1 - e^{-t/25\,x10^{--6}}) = 2x(1 - e^{-t/25\,x10^{-6}})$ mA

- The current in the circuit @100μsec is given by
  $i(t)$ @100 μsec $= 2x\,(1 - e^{-100\,\mu sec/25\,\mu sec})$ mA $= 2x(1 - e^{-4})$ mA = 1.9633 mA

i(t) @100 μsec = 1.9633 m

- The Voltage across the resistoris given by vR@100 μsec = R x i(t) @100 μsec
  vR@100 μsec = 8 KΩ x1.9633 mA = 15.707 V

vR@100 μsec = 15.707 V

(b) (b)

- The current in the circuit @350 μsec is the decaying current and is given by:

$i(t)$Decaying= $I(0).e^{-t/\tau}$ where I(0) is the initial current and in this case it is the growing current @250μsec. (Since the switch is changed @250μsec) The time t is to be reckoned from this time of 250 μsec. Hence t = (350—250) = 100μsec. So we have to calculate first i(t)growing(@250 μsec)which is given by:
$i(t)$ growing(@250 μsec) $= V/R\,(1 - e^{-t/\tau}) = (16/8)X10^{--3}\,(1 - e^{-t/25\,\mu sec}) = 2x(1 - e^{-250/25\,\mu sec})$ mA
$=2x(1 - e^{-10})$ mA = 1.999 mA

i(t)growing(@250 μsec)= 1.999 mA = I(0)

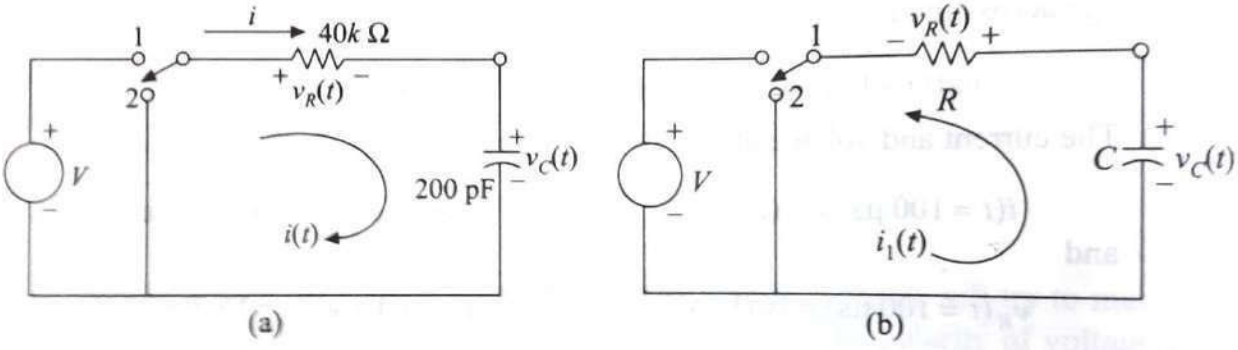
Hence i(t)@350 μsec $=I(0).e^{-t/\tau} = 1.99x\,e^{-100\,\mu sec/25\,\mu sec}$mA $= 1.99x\,e^{-4}$mA = 0.03663 mA

i(t)@350 μsec = 0.03663 mA

- The voltage across the resistor vR @350 μsec = Rxi(t@350 μsec) = 8KΩx0.03663 mA
  vR @350 μsec= 0.293V

Example 6: In the circuit shown below the switch is kept in position 1 up to 100 μ secs and then it is moved to position 2 . Supply voltage is 5V DC . Find

a) The current and voltage across the capacitor at t = 40 μ secs
b) The current and voltage across the resistor at t = 150 μ secs



Solution:The time constant τ of the circuit is same in both conditions and is given by τ = RC = $40 \times 10^{3} \times 200 \times 10 \times^{-12}$ = 8 μsec

a) The time t = 40 μsec corresponds to the switch in position 1 and in that condition the current i(t) is given by the standard expression for charging current

$$i(t) = (V/R)\,[e^{-t/\tau}]$$

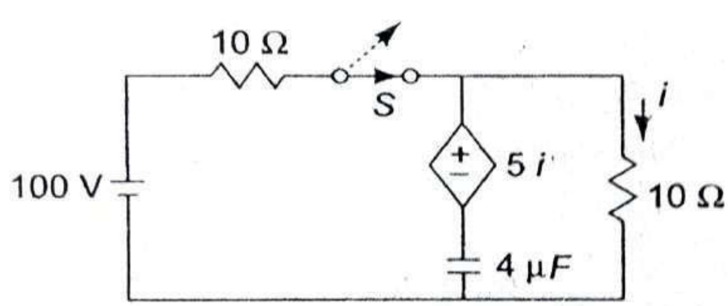
i(t) @40 μsec = 5v/40KΩ $[e^{-40/8}]$ Amps = 0.125x$[e^{-5}]$ mA = 0.84224 μA

i(t) @40 μsec = 0.84224 μA

The voltage across the capacitor during the charging period is given by V $[1- e^{-t/\tau}]$. vC(t) @40 μsec = 5[1 -- $e^{-40/8}$] = 5[1 -- e^{-5}] = 4.9663 Volts

vC(t) @40 μsec = 4.9663 Volts

Example 9 : In the circuit shown below find an expression for the current i(t) when the switch is opened at time t= 0



Solution: The following points may be noted with reference to this circuit:

- When the switch is opened the circuit is equivalent to a normal source free circuit but with a current dependent voltage source given as 5i.
- The initial current I_0 when the switch is opened is same as the current when the switch was closed for a long time and is given by $I_0 = 100/(10+10) = 5$ Amps

The loop equation when the switch is opened is given by :

$$(1/4x10^{-6})\int idt + 10i = 5i$$
$$(1/4x10^{-6})\int idt + 5i = 0$$

Differentiating the above equation we get :

$$5.(di/dt) + (1/4x10^{-6})i = 0 \text{ i.e. } = (di/dt) + (1/20 \times 10^{-6})i = 0$$

Writing the above equation in the 's'notation where 's' is the operator equivalent to (d/dt) we get

$(s+ 1/20 \times 10^{-6}) i = 0$ and the characteristic equation will be $(s+ 1/20 \times 10^{-6}) = 0$

The solution i(t) is given by $i(t) = K . e^{-t/20 \times 10^{-6}}$. The constant K can be evaluated by invoking the initial condition that i(t) @ t=0is equal to $I_0 = 5$ amps .Then the above equation becomes:

$5 = K . e^{-t/20 \times 10^{-6}}$ i.e K = 5 and hence the current in the circuit when the switch is opened becomes: $i(t) = 5. e^{-t/20 \times 10^{-6}}$ Amps

Example 10: A series RLC circuit as shown in the figure below has R = 5Ω,L= 2H and C = 0.5F.The supply voltage is 10 V DC . Find

a) The current in the circuit when there is no initial charge on the capacitor.
b) The current in the circuit when the capacitor has initial voltage of 5V
c) Repeat question (a) when the resistance is changed to 4 Ω

Where $v_C(t)_0$ is the initial capacitor voltage when the switch was changed to position 2 and it is the voltage that has built up by 100 μsec during the charging time (switch in position 1) and hence is given by

$v_C(t)@100\mu sec = 5[1- e^{-100/8}]$ volts $= 5x[1- e^{-12.5}]$ Volts $= 4.999$ Volts

And now t=150 μsec from beginning is equal to t = (150-100) = 50 μsec from the time switch is changed to position 2.

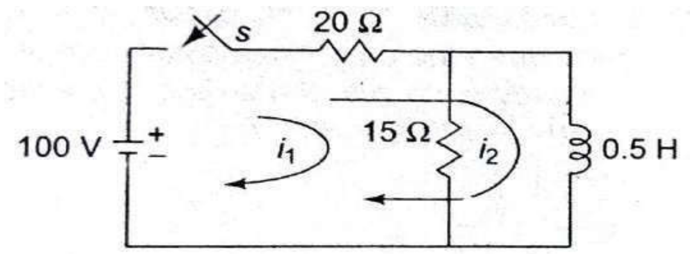
Therefore the current through the resistor at 150 μsec from the beginning = $i(t)_{150\mu sec} = (4.999/40K\Omega). e^{-t/\tau}$

$i(t)_{150\mu sec} = 0.1249 \times e^{-50/8} = 0.241\ \mu A$

$$i(t)_{150\mu sec} = 0.241\ \mu A$$

And the voltage across the resistor = R x i(t) = 40KΩ x 0.241 μA = 0.00964v

Example 7: In the circuit shown below find out the expressions for the current i1 and i2 when the switch is closed at time t= 0



Solution: It is to be noted that in this circuit there are two current loops 1 and 2 . Current i1 alone flows through the resistor 15 Ω and the current i2 alone flows through the inductance0.5 H where as both currents i1 and i2 flow through the resistor 20 Ω. Applying KVL to the two loops taking care of this point we get

20(i1 + i2) + 15 i1 = 100 i.e ---------------------------35 i1 + 20 i2 = 100 (1)

and 20(i1 + i2) + 0.5 di2/dt = 100 ; 20 i1+ 20 i2 +0.5 di2/dt = 100-- (2)

Substituting the value of i1 = [100/35 – (20/35) i2] = 2.86 – 0.57 i2 obtained from the above equation (1) into equation (2) we get :

20 [2.86 – 0.57 i2] +20i2 + 0.5 (di2/dt) = 100

57.14 – 11.4 i2 +20i2 + 0.5 (di2/dt) = 100
(di2/dt) i2 +17.14 i2 = 85.7

The solution for this equation is given by $i2(t) = K. e^{-17.14t} + 85.72/17.14$ and the constant K can be evaluated by invoking the initial condition. The initial current through the inductor = 0 at time t = 0 .

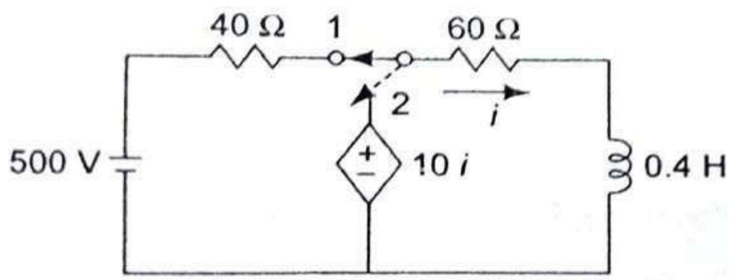
Hence K = -- 85.72/17.14 = -- 5

Therefore $i2(t) = 5 (1-- e^{-17.14t})$ Amps

And current $i1(t) = 2.86 – 0.57\ i2 = 2.86 – 0.57 [5 (1-- e^{-17.14t})] = 0.01 + 2.85\ e^{-17.14t}$ Amps

And current $i1(t) = 0.01 + 2.85\ e^{-17.14t}$ Amps

Example 8 : In the circuit shown below find an expression for the current i(t) when the switch is changed from position 1 to 2 at time t= 0 .



Solution: The following points are to be noted with reference to this circuit:

- When the switch is changed to position 2 the circuit is equivalent to a normal source free circuit but with a current dependent voltage source given as 10i.
- The initial current in position 2 is same as the current when the switch was in position 1 (for a long time) and is given by $I_0 = 500/(40+60) = 5$ Amps

The loop equation in position 2 is given by : 60i + 0.4 di/dt = 10i i.e (50/0.4)i + di/dt = 0

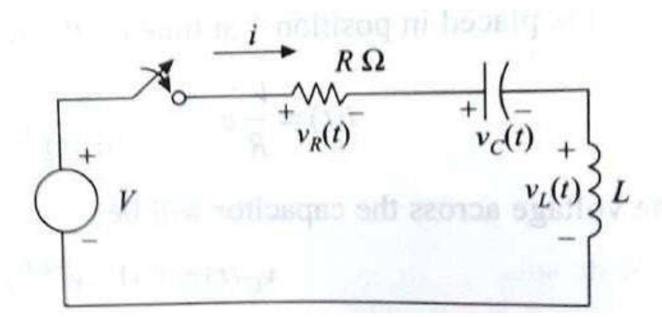
Writing the equation in the 's'notation where 's' is the operator equivalent to (d/dt) we get (

s+ 125) i = 0 and the characteristic equation will be (s+ 125) = 0

Hence the solution i(t) is given by $i(t) = K . e^{--125t}$. The constant K can be evaluated by invoking the initial condition that i(t) @ t=0 is equal to I_0 = 5 amps .Then the above equation becomes:

$5 = K . e^{—125X0}$ i.e K = 5 and hence the current in the circuit when the switch is changed to position 2 becomes: $i(t) = 5. e^{--125t}$ Amps

d) Repeat question (a) when the resistance is changed to 1 Ω



Solution: The basic governing equation of this series circuit is given by :

$$Ri + 1/C\int \square\square\square + L.\,(di/dt)= V$$

On differentiation we get the same equation in the standard differential equation form

$$L(d^2i/dt^2)+ R(di/dt)+ (1/C)i = 0$$

By dividing the equation by L and using the operator ‘s’ for d/dt we get theequation in the form of characteristic equation as :

$$[s^2 + (R/L)s + (1/LC)] = 0$$

Whose roots are given by:

$$s1,s2 =- R/2L\pm\sqrt{[(R/2L)^2- (1/LC)]}= -\alpha \pm\sqrt{(\alpha^2- \omega0^2)}$$

and three types of solutions are possible.

1. $\alpha > \omega0$, i.e when $LC > (2L/R)^2$ s1 and s2 will both be negative real numbers, leading to what is referred to as an *over damped response*given by :
$$i\,(t) = A1e^{s1t}+ A2e^{s2t}$$
2. $\alpha = \omega0$, , i.e when $LC = (2L/R)^2$ s1 and s2 are equal which leads to what is called a *critically damped response*given by :
$$i\,(t) = e^{-\alpha t}(A1t + A2)$$
3. $\alpha < \omega0$, i.e when $LC < (2L/R)^2$ both s1 and s2 will have nonzero imaginary components, leading to what is known as an *under damped response*given by :

$$i\,(t) = e^{-\alpha t}(A1\ \cos\omega d\ t + A2\ \sin\omega d\ t)$$

where$\omega d$is called *natural resonant frequency* and is given given by:

$$\omega d = \sqrt{\omega0^2- \alpha^2}$$

The procedure to evaluate the complete solution consists of the following steps for each part of the question:

1. We have to first calculate the roots for each part of the question and depending on to which case the roots belong we have to take the appropriate solution.
2. Then by invoking the first initial condition i.e i = 0 at t=0 obtain the first relation between A1 and A2or one of its values.
3. If one constant value is obtained directly substitute it into the above solution and take its first derivative. Or else directly take the first derivative of the above solution

4. Now obtain the value di/dt @ t= 0from the basic RLC circuit equation by invoking the initial conditions of vC@ t=0 and i(t) @ t=0 . Now equate this to the differential of the solution for i(t) to get the second relation between A1 and A2(or the second constant . Now using these two equations we can solve for A1 and A2 and subsititute in the solution for i(t) to get the final solution.

(a) $s1,s2 = -R/2L \pm \sqrt{[(R/2L)^2 - (1/LC)]} = (-5/2x2) \pm \sqrt{[(5/2x2)^2 - (1/2x0.5)]} = -1.25 \pm 0.75$.
i.e. s1 = --0.5 and s2 = --2

In this case the roots are negative real numbers and the solution is
given by : $i(t) = A1e^{s1t} + A2e^{s2t} = A1e^{-0.5t} + A2e^{-2t}$ (1)
Now we will apply the first initial condition i.e i(t) = 0 at t=0 .Then we get
$0 = A1e^{-0.5x0} + A2e^{-2x0}$ i.e. A1+ A2 = 0

The basic equation for voltage in the series RLC circuit is

given as : V = R.i(t) + vC(t) + L.(di/dt) i.e di/dt = 1/L [V -

R.i(t) - vC(t)
At time t=0 we get
(di/dt)@ t=0 ---= 1/L [V -R.i(t=0) - vC(t=0)]
(2)
But we know that the voltage across the capacitor and current are zero at time t=0 .
Therefore (di/dt)@ t=0 = V/L = 10/2 = 5 ------- (3)

Now the equation for i(t) at equation (1) is differentiated to get
$(di/dt) = -0.5A1e^{-0.5t} - 2A2e^{-2t}$
and the above value of (di/dt)@ t=0 = 5 is substituted in that to get the second equation with A1 and A2
(di/dt)@ t=0 $= 5 = -0.5A1e^{-0.5x0} - 2A2e^{-2x0} = -0.5A1 \quad 2A2$
Now we can solve the two equations for A1 and A2

A1+ A2 = 0 and -0.5A1--2A2 = 5 to get A1 = 10/3 and A2 = 10/3

And the final solution for i(t) is : $(10/3)[e^{-0.5t} - e^{-2t}]$ Amps

(b) At time t=0 the voltage across the capacitor = 5V ie. vC(t=0) = 5V . But i(t=0) is still =0.using these values in the equation (2) above we get
(di/dt)@ t=0 = ½ (10-5) = 2-5
Then the two equations in A1 and A2 are A1+ A2 = 0 and -0.5A1--2A2 =2.5 Solving these two equations we get A1 = 5/3 and A2 = -5/3

And the final solution for i(t) is : $(5/3)[e^{-0.5t} - e^{-2t}]$ Amps

(c) The roots of the characteristic equation when the Resistance is changed to 4 $s1,s2 = -R/2L \pm \sqrt{[(R/2L)^2 - (1/LC)]} = (-4/2x2) \pm \sqrt{[(4/2x2)^2 - (1/2x0.5)]} = -1.0$
i.e the roots are real and equal and the solution is given by

$$i(t) = e^{-\alpha t}(A1t + A2) = e^{-1t}(A1t + A2)\text{-}(4)$$

Now using the initial condition i(t) = 0 at time t=0 we get A2 = 0

We have already found in equation (3) for the basic series RLC circuit (di/dt)@ t=0 = 5Now we will find di(t)/dt of equation (4) and equate it to the above value. di /dt = $-e^{-1t}(A1t + A2) + e^{-1t}(A1) = e^{-1t}[A1 - A1t - A2]$ and

(di /dt) @t=0= e^{-1x0} [A1 – A1x0 –A2] i.e A1 – A2 = 5 Therefore A1 =5 and A2 = 0

And the final solution for i(t) is $i(t) = 5te^{-1t}$Amps

(d) Roots of the characteristic equation when the resistance is changed to 1 Ω are :

$s1,s2 = -R/2L \pm \sqrt{[(R/2L)^2 - (1/LC)]} = (-1/2x2) \pm \sqrt{[(1/4)^2 - (1/2x0.5)]} = --0.25 \pm j0.94$

The roots are complex and so the solution is then given by : $i(t) = e^{-\alpha t}(A1 \cos\omega d\, t + A2 \sin\omega d\, t)$
Where α = 0.25 and ωd= 0.9465

Now we will apply the initial conditions to find out the constants A1 and A2

First initial condition is i(t)@t=0 = 0 applying this into the equation : $i(t) = e^{-\alpha t}(A1 \cos\omega d\, t + A2 \sin\omega d\, t)$ we get A1 = 0 and using this value of A1 in the abve equation for i(t) we get $i(t) = e^{-\alpha t}(A2 \sin\omega d\, t)$

We have already obtained the second initial condition as di (t) /dt@t=0= 5 from the basic equation of the series RLC circuit. Now let us differentiate above equation for current i.e :$i(t) = e^{-\alpha t}(A2 \sin\omega d\, t)$ and equate it to 5 to get the second constant A2

di (t) /dt = $e^{-\alpha t}(A2\, \omega d \cos \omega d\, t) + (A2 \sin\omega d\, t)$

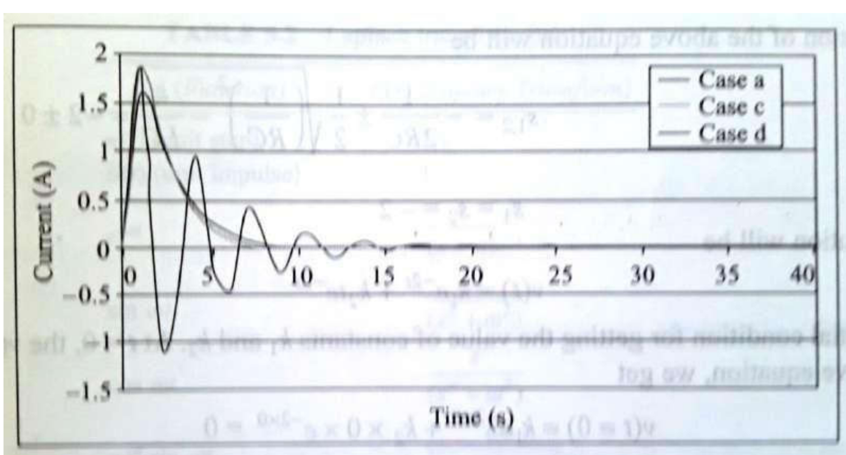
. $-\alpha.\ e^{-\alpha t}$ di (t) /dt @t=0 =A2. ωd= 5

i.e A2 = 5 / ωd = 5/0.94 = 5.3

Now using this value of A2 and the values of α = 0.25 and ωd = 0.94in the above expression for the current we finally get :

$$i(t) = e^{-0.25t}(2.569 \sin 1.9465t)$$

The currents in all the three different cases (a), (c) and (d) are shown below



MODULE-V

Network Synthesis

Hurwitz Polynomial:

A polynomial p(s) is said to be Hurwitz if all the roots of p(s) are located in the open left half (LH) s-plane (not including the imaginary axis).

Let p(s) be the polynomial in question. Assume first that p(s) is neither an even nor an odd polynomial. To test whether such a polynomial p(s) is indeed a Hurwitz polynomial, we may use the Hurwitz test.

- First decompose p(s) into its even and odd parts, M(s) and N(s),respectively, as p(s) = M(s) +N(s).

Using M(s) and N(s) we form the test ratio T(s), whose numerator has higher degree than that of its denominator. Suppose that p(s) is a polynomial of degree d. Then Let p(s) be the polynomial in question. Assume first that p(s) is neither an even nor an odd polynomial. To test whether such a polynomial p(s) is indeed a Hurwitz polynomial, we may use *the Hurwitz test*.

- First decompose p(s) into its even and odd parts, M(s) and N(s),respectively, as p(s) = M(s) +N(s).
- Using M(s) and N(s) we form the test ratio T(s), whose numerator has higher degree than that of its denominator. Suppose that p(s) is a polynomial of degree d. Then

$$T(s) = \frac{N(s)}{M(s)} \quad \text{if } d \text{ is an odd integer} \qquad (4\text{-}8a)$$

$$T(s) = \frac{M(s)}{N(s)} \quad \text{if } d \text{ is an even integer} \qquad (4\text{-}8b)$$

- Next, we perform the continued fraction expansion about infinity on the testratio T(s), removing one pole at a time in the form of a quotient qs, resulting in:

$$T(s) = q_1 s + \cfrac{1}{q_2 s + \cfrac{1}{q_3 s + \cfrac{1}{\ddots + \cfrac{1}{q_{\hat{d}} s}}}} \qquad (4\text{-}9)$$

Where $q_i s$ is the ith quotient, and q_i, is the associated coefficient.

- If there is one or more quotients with negative coefficients, then p(s) isneither a Hurwitz nor a modified Hurwitz polynomial.
- On the other hand, if there are d quotients ($d = \hat{d}$) and every quotient has apositive coefficient, then p(s) is a Hurwitz polynomial.
- Finally, if the number of quotient $\hat{d}$ is less than d but every quotient has apositive coefficient, this means that there is a common factor k(s) between M(s) and N(s). Hence, we can write p(s)as:

$$p(s) = k(s)\,[\hat{M}(s) + \hat{N}(s)] = k(s)\hat{p}(s) \qquad (4\text{-}10)$$

where $M(s) = k(s)\hat{M}(s)$, $N(s) = k(s)\hat{N}(s)$, and $\hat{p}(s) = \hat{M}(s) + \hat{N}(s)$.

Because all the $\hat{d}$ quotients of T(s) have positive coefficients, the polyno- mial p(s) in (4-10) is Hurwitz. Thus, if k(s) is a modified Hurwitz polynomial [i.e., if all the roots of k(s) are simple and purely imaginary], then p(s) is a modified Hurwitz polynomial.

- A procedure to determine if k(s) is a modified Hurwitz polynomial isdescribed in the following in conjunction with the case when p(s) is either an even or an odd polynomial.
- Suppose now that p(s) is either an even or an odd polynomial of degree d. is amodified Hurwitz polynomial if and only if p(s) has only simple and imaginary axis roots (including the origin).
- To determine if p(s) is a modified Hurwitz polynomial, we form atest ratio

$\hat{T}(s)$:

$$\hat{T}(s) = \frac{p(s)}{(d/ds)\,p(s)} = \frac{p(s)}{p'(s)} \qquad (4\text{-}12)$$

And perform the continued fraction expansion about infinity on $\hat{T}(s)$, as in (4- 9). Then p(s) is a modified Hurwitz polynomial if and only if there are d quotients in the expansion and each quotient has a positive coefficient.

- In the case when p(s) is either an even or an odd polynomial, if there is one ormore negative coefficient in the continued fraction expansion of T^(s), then p(s) has a RH s-plane root; and if all coefficients are positive but there are only d^ < d quotients, then all roots of p(s) are on the imaginary axis of the s-plane, but p(s) has non-simple or multiple roots. Either situation implies that p(s) is not a modified Hurwitz polynomial.

Example **Determine if**

$$p(s) = s^4 + 3s^3 + 5s^2 + 5s + 2 \tag{4-13}$$

is a Hurwitz polynomial.

or

$$T_2(s) = \frac{(10/3)s^2 + 2}{(16/5)s} \tag{4-19}$$

Clearly, $T_2(\infty) = \infty$. Removing the pole at infinity from $T_2(s)$, we obtain

$$T_2(s) = \frac{25}{24}s + \frac{1}{T_3(s)} \tag{4-20}$$

where $(25/24)s$ is the third quotient, 25/24 is its coefficient, and

$$\frac{1}{T_3(s)} = T_2(s) - \frac{25}{24}s = \frac{2}{(16/5)s} = \frac{1}{(8/5)s} \tag{4-21}$$

is the third remainder. Substituting (4-20) and (4-21) into (4-18), we obtain the continued fraction expansion of $T(s)$ at $s = \infty$ as

$$T(s) = \frac{1}{3}s + \cfrac{1}{(9/10)s + \cfrac{1}{(25/24)s + \cfrac{1}{(8/5)s}}} \tag{4-22}$$

Because there are four quotients and their coefficients are positive (being 1/3, 9/10, 25/24, and 8/5), $p(s)$ is Hurwitz.

Routh–Hurwitz stability criterion:

A tabular method can be used to determine the stability when the roots of a higher order characteristic polynomial are difficult to obtain. For an *n*th-degree polynomial

$$D(s) = a_n s^n + a_{n-1}s^{n-1} + \cdots + a_1 s + a_0$$

the table has *n* + 1 rows and the following structure:

a_n	a_{n-2}	a_{n-4}	...

a_{n-1}	a_{n-3}	a_{n-5}	$\cdots$
b_1	b_2	b_3	$\cdots$
c_1	c_2	c_3	$\cdots$
$\vdots$	$\vdots$	$\vdots$	$\ddots$

Where the elements b_i and c_i can be computed as follows:

$$b_i = \frac{a_{n-1} \times a_{n-2i} - a_n \times a_{n-2i-1}}{a_{n-1}}.$$

$$c_i = \frac{b_1 \times a_{n-2i-1} - a_{n-1} \times b_{i+1}}{b_1}.$$

When completed, the number of sign changes in the first column will be the number of non- negative poles.

In the first column, there are two sign changes, thus there are two non-negative roots where the system is unstable. Sometimes the presence of poles on the imaginary axis creates a situation of marginal stability. The row of polynomial which is just above the row containing the zeroes is called "Auxiliary Polynomial".

$$s^6 + 2s^5 + 8s^4 + 12s^3 + 20s^2 + 16s + 16 = 0.$$

We have the following table:

1	8	20	16
2	12	16	0
2	12	16	0
0	0	0	0

In such a case the Auxiliary polynomial is $A(s) = 2s^4 + 12s^2 + 16.$ which is again equal to zero. The next step is to differentiate the above equation which yields the following polynomial. $B(s) = 8s^3 + 24s^1.$. The coefficients of the row containing zero now become "8" and "24". The process of Routh array is preceded using these values which yield two points on the imaginary axis. These two points on the imaginary axis are the prime cause of marginal stability.

Properties of Positive Real Function:

- The sum of two PR functions is PR.
- The composition of two PR functions is PR. In particular, if $Z(s)$ is PR, then so are $1/Z(s)$ and $Z(1/s)$.

- All the poles and zeros of a PR function are in the left half plane or on its boundary the imaginary axis.

- Any poles and zeroes on the imaginary axis are simple (have a multiplicity of one).
- Any poles on the imaginary axis have real strictly positive residues, and similarly at any zeroes on the imaginary axis, the function has a real strictly positive derivative.
- Over the right half plane, the minimum value of the real part of a PR function occurs on the imaginary axis (because the real part of an analytic function constitutes harmonic over the plane, and therefore satisfies the maximum principle).
- For a rational PR function, the number of poles and number of zeroes differ by at most one

LC Network Synthesis

If a network contains only inductors and capacitors, it is called a pure reactive network. In pure reactive network, the average power dissipated is zero so that it is called lossless network. Therefore, the real part of the impedance/admittance function is zero for pure imaginary frequency, $s = j\omega$. Consider a deriving-point impedance function Z(s).

Re[Z(jω)] = 0

Let Z(s) be written as follows:

$$Z\ (s) = \frac{M_1(s) + N_1(s)}{M_2(s) + N_2(s)}$$

Where, M and N are even and odd parts respectively.

Re[Z(jω)] = 0

- $M1(j\omega)M2(j\omega) - N1(j\omega)N2(j\omega) = 0$
- $M1 = 0 = N2$ *Or* $M2 = 0 = N1$

$$Z(s) = \frac{N_1}{M_2} \text{ or } \frac{M_1}{N_2}$$

- *Z(s)* is always even to odd (*Ne(s)/Do(s)*) or odd to even (*No(s)/De(s)*) quotient ofpolynomials.
- Since N and D are either even or odd polynomials, all poles and zeros of Z lie on the jω – axis
- Z has pole at zero (when *Z(s) = Ne(s)/Do(s)*) or zero at zero (when *Z(s) =No(s)/De(s)*)
- Degrees of N and D differ exactly byone.
- Z has pole at infinity (if Deg N > Deg D) or zero at infinity (if

Deg N < DegD) In general, Z(s) can be written asfollows:

$$Z(s) = \frac{H(s^2+\omega_{z1}^2)(s^2+\omega_{z2}^2)\ldots}{s(s^2+\omega_{p1}^2)(s^2+\omega_{p2}^2)\ldots}$$

$\omega_{z1} = 0$ if Z has zero at s

= 0 The general partial fraction expansion will be:

$$Z(s) = \frac{k_0}{s} + \frac{k_1 s}{s^2 + \omega_{p1}^2} + \frac{k_2 s}{s^2 + \omega_{p2}^2} + \ \ldots + Hs$$

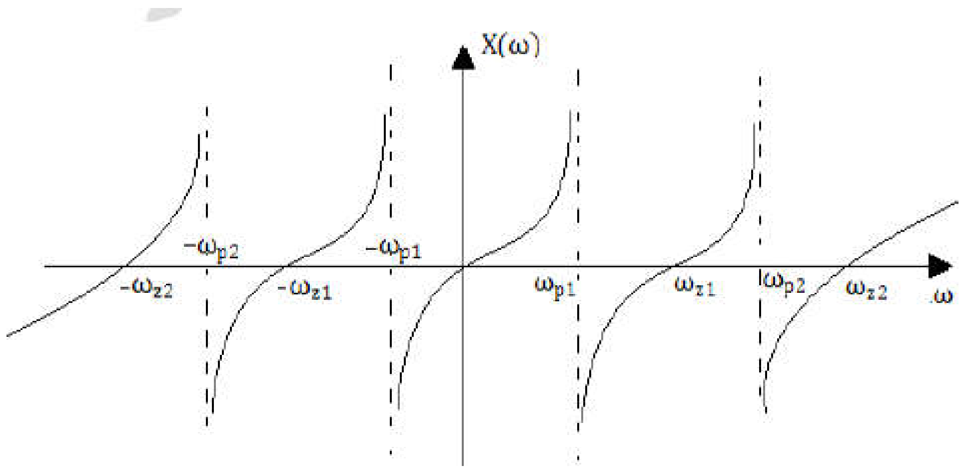
The first term exists Z has pole at s = 0, and the last term exists if Z has pole at infinity. Since *Re[Z(jw)]*

= 0

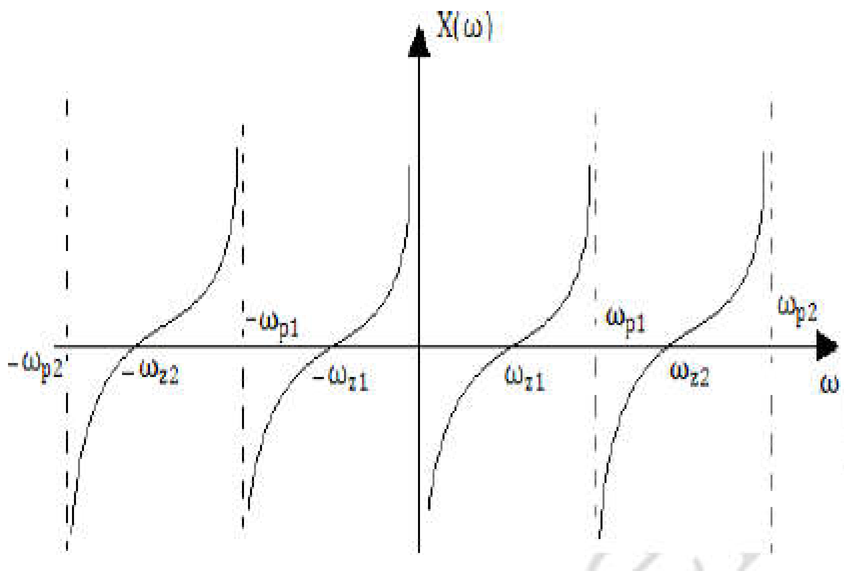
Z(jω) = jX(ω) the imaginary part X(ω) is called the reactance function.

X (ω) is an odd function.

X (ω) has a positive slope where the minimum slope is H. Therefore, poles and zeros are interlaced.



a) When Z has zero at s =0



Definition:

- Poles and zeros of Z(s) are collectively known as critical frequencies
- Poles and zeros located at zero and infinity are called external critical frequencies.
- All other poles and zeros are known as internal

critical frequencies. Summary:

Z(s)

- Has only simple poles and zeros all interlaced on the jω –axis.
- Is a quotient of even and odd polynomials? (Ne(s)/Do(s) or No(s)/De(s))

- has a zero (No/De) or pole (Ne/Do) at s = 0
- has a zero (Deg N < Deg D) or pole (Deg N > Deg D) at infinity
- can be completely and uniquely specified by its internal critical frequencies

and H All above properties also apply for the deriving-point admittance function

Y(s) **Realization of LC Networks:**

There are four canonical forms:

¬ Foster Forms

- Foster-1: partial fraction expansion at poles ofZ(s)
- Foster-2: partial fraction expansion at poles ofY(s)

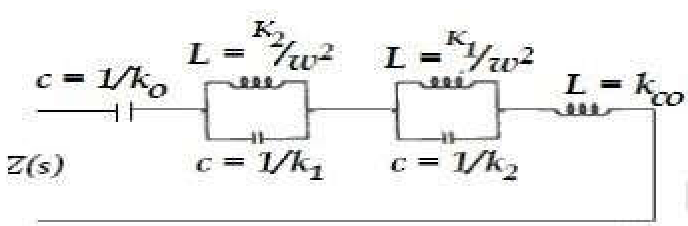
¬ Cauer Forms

- Cauer-1: Continued fraction expansion about infinity (successive removal of pole atinfinity)
- Cauer-2: Continued fraction expansion about the origin (successive removal of pole atzero)

Foster Realizations:

Foster-1:

$$Z(s) = \frac{k_0}{s} + \frac{k_1 s}{s^2 + \omega_{p1}^2} + \frac{k_2 s}{s^2 + \omega_{p2}^2} + \ldots + k_\infty s$$



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Realization of LC Networks:

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<u>Foster-1:</u>

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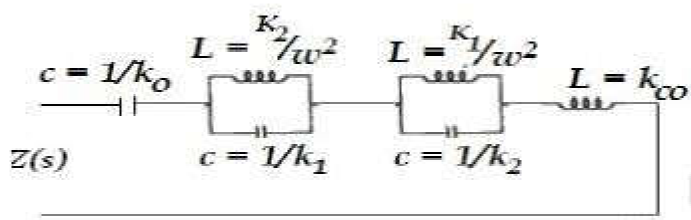
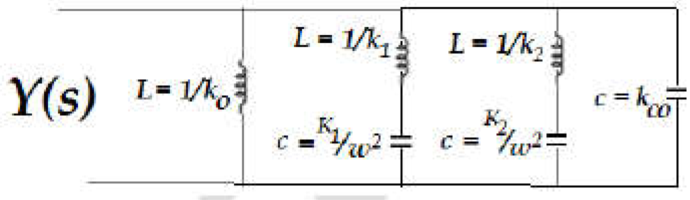


$$Y(s) = \frac{k_0}{s} + \frac{k_1 s}{s^2 + \omega_{p1}^2} + \frac{k_2 s}{s^2 + \omega_{p2}^2} + \dots + k_\infty s$$



<u>Foster-2:</u>

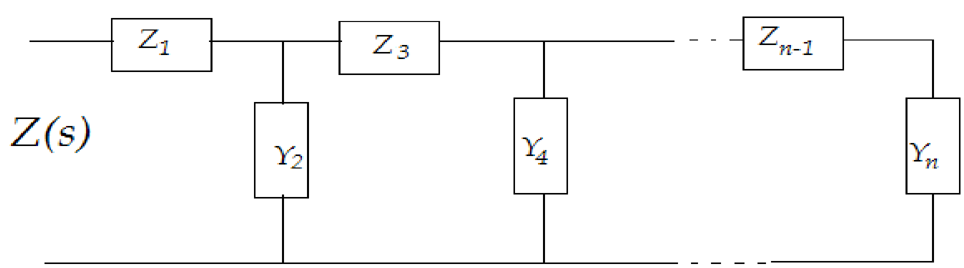
Cauer Realizations

Removal of pole at zero or infinity leaves a remainder function that has zero at zero or infinity respectively. This zero can be removed as a pole of the reciprocal function. Similarly, removal of pole from the reciprocal leaves another remainder function that has zero at that frequency. Repeated application of this process gives continued fraction expansion of the deriving point impedance or admittance function. This expansion can be realized as a ladder network.

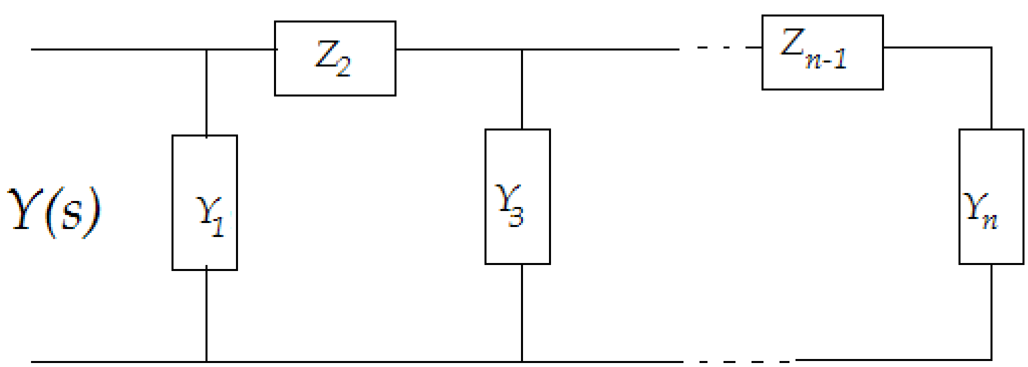
If the deriving-point function is H(s), its PFE will be:

$$H(s) = q_1(s) + \cfrac{1}{q_2(s) + \cfrac{1}{q_3(s) + \cfrac{1}{q_4(s) + \cdots}}}$$

If H(s) impedance function



If H(s) is admittance function



Cauer- 1: obtained by continued removal of pole at infinity. Cauer-1 form of H(s) will be:

$$H(s) = \alpha_1 s + \cfrac{1}{\alpha_2 s + \cfrac{1}{\alpha_3 s + \cfrac{1}{\alpha_4 s + \cdots}}}$$

Cauer- 2: obtained by continued removal of pole at zero. Cauer-2 form of H(s) will be:

$$H(s) = \frac{\beta_1}{s} + \cfrac{1}{\frac{\beta_2}{s} + \cfrac{1}{\frac{\beta_3}{s} + \cfrac{1}{\frac{\beta_4}{s} + \cdots}}}$$

Example 2

Given the following LC impedance function.

$$Z(s) = \frac{s^3 + 2s}{s^4 + 4s^2 + 3}$$

a) Plot$X(\omega)$

b) Find the two Foster and two Cauer realizations of$Z(s)$.

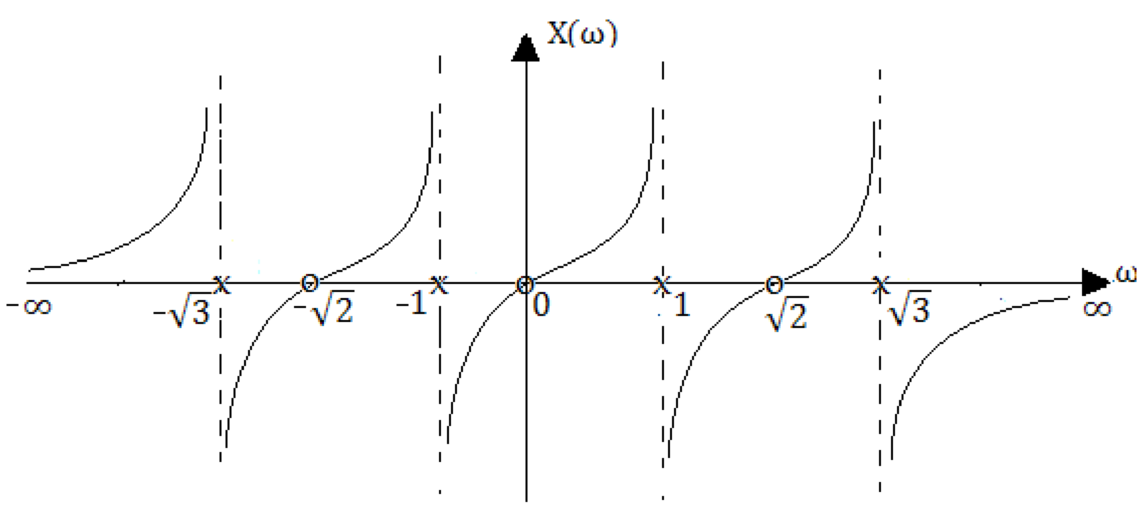
Solution:

a)

$$Z(s) = \frac{s(s^2 + 2)}{(s^2 + 1)(s^2 + 3)}$$

$$X(\omega) = \frac{1}{j} Z(j\omega) = \frac{\omega(2 - \omega^2)}{(1 - \omega^2)(3 - \omega^2)}$$

Poles: ω = 1,
√3 Zeros: ω =
0, √2,



∞ H = 1

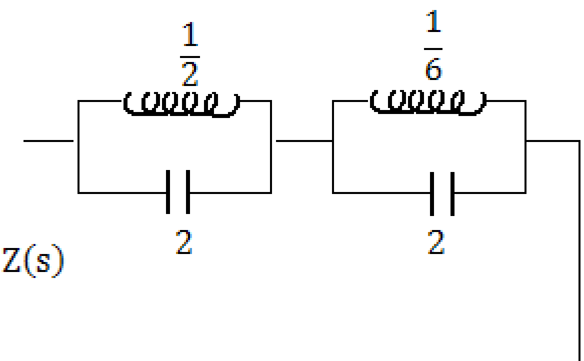
b)

Foster –1

$$Z(s) = \frac{s(s^2+2)}{(s^2+1)(s^2+3)}$$

$$= \frac{\frac{1}{2}s}{s^2+1} + \frac{\frac{1}{2}s}{s^2+3}$$

$$= \frac{1}{2s + \frac{1}{\frac{1}{2}s}} + \frac{1}{2s + \frac{1}{\frac{1}{6}s}}$$

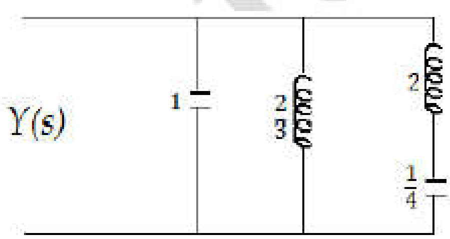


Foster –2

$$Y(s) = \frac{1}{Z(s)} = \frac{(s^2+1)(s^2+3)}{s(s^2+2)}$$

$$= s + \frac{\frac{3}{2}}{s} + \frac{\frac{1}{2}s}{s^2+2}$$

$$= s + \frac{1}{\frac{2}{3}s} + \frac{1}{2s + \frac{1}{\frac{1}{4}s}}$$



Cauer – 1 pole removal at infinity

$$Z(s) = \frac{s^3 + 2s}{s^4 + 4s^2 + 3}$$

Note: Z(s) does not have pole at infinity. Therefore, we take Y(s) since it will have pole at infinity.

$$Y(s) = \frac{1}{Z(s)} = \frac{(s^2+1)(s^2+3)}{s(s^2+2)}$$

$$= \frac{s^4 + 4s^2 + 3}{s^3 + 2s}$$

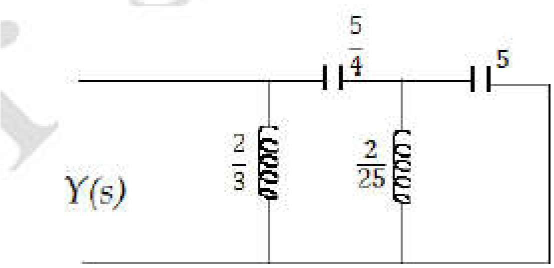
¬ Cauer – 2 Pole removals atzero

$$Z(s) = \frac{s^3 + 2s}{s^4 + 4s^2 + 3}$$

Z(s) does not have pole at the origin. Therefore, we take Y(s) since it will have pole at zero.

$$Y(s) = \frac{1}{Z(s)} = \frac{3 + 4s^2 + s^4}{2s + s^3}$$

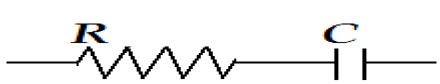
$$= \frac{3}{2s} + \cfrac{1}{\cfrac{4}{5s} + \cfrac{1}{\cfrac{25}{2s} + \cfrac{1}{\cfrac{1}{5s}}}}$$



RC and RL Networks

RC Networks

An RC-network is built from resistors and capacitors so that it can be taken as interconnection of smaller networks shown below.Where R = 0 (short circuited) if the sub network contains only a



capacitor and c = ∞ (short circuited) if the sub network contains only a resistor. Therefore, the general impedance function of a sub network can be written as:

$Z_{RC}(s) = R_i + 1/C_i s = A + 1/Bs$

Similarly, an LC-network is built from sub networks of the form shown below.

Where L = 0 (short circuited) when there is no inductor and C = ∞ when there is no capacitor. The general impedance function of such sub network can be written as:

$Z_{LC}(s) = L_i s + 1/C_i s = \alpha s + 1/\beta s$

Let us substitute *s* with *p*

$Z_{LC}(p) = \alpha p + 1/\beta p$

$$\frac{1}{p} Z_{LC}(p) = \alpha + \frac{1}{\beta p^2}$$

$$[\frac{1}{p} Z_{LC}(p)]_{p^2=s} = \alpha + \frac{1}{\beta s}$$

This transformed function has the same form as the RC-network impedance function so that the general RC-network impedance function can be obtained from the general LC-network

$$Z_{RC}(s) = [\frac{1}{p} Z_{LC}(p)]_{p^2=s}$$

impedance function.

Similarly, the admittance function can be obtained from reciprocal of the impedance function.

$$Y_{RC}(s) = \frac{1}{Z_{RC}(s)}$$

$$= [p \frac{1}{Z_{LC}(p)}]_{p^2=s}$$

$$Y_{RC}(s) = [pY_{LC}(p)]_{p^2=s}$$

Case 1: Z_{LC} has pole at s = 0

$$Z_{LC}(s) = \frac{N(s)}{D(s)} = \frac{H(s^2 + \omega_{z1}^2)(s^2 + \omega_{z2}^2)\ldots}{s(s^2 + \omega_{p1}^2)(s^2 + \omega_{p2}^2)\ldots} \quad \text{where } 0 < \omega_{z1} < \omega_{p1} < \omega_{z2} < \omega_{p2} < \cdots$$

a) When Deg N = Deg D +1

- Pole at∞
- Highest critical frequency is a pole at infinity
- Highest internal critical frequency is a zero

b) When Deg N +1 = Deg D

- Highest critical frequency is a zero at infinity
- Highest internal critical frequency is a pole

$$Z_{RC}(s) = [\frac{1}{p} Z_{LC}(p)]_{p^2=s} = \frac{N_2(s)}{D_2(s)} = \frac{H(s + \omega_{z1}^2)(s + \omega_{z2}^2)\ldots}{s(s + \omega_{p1}^2)(s + \omega_{p2}^2)\ldots}$$

$$= \frac{H(s + \alpha_1)(s + \alpha_2)\ldots}{s(s + \beta_1)(s + \beta_2)\ldots} \quad where\ 0 < \alpha_1 < \beta_1 < \alpha_2 < \beta_2 < \cdots$$

The first critical frequency is a pole at zero

► Lowest critical frequency is apole

Poles and zeros are simple and lie on the negative real axis

► Poles and zerosinterlace

a) When Deg N = Deg D +1

- Deg N2 = DegD2
- No pole atinfinity
- Highest critical frequency for ZRC is azero

b) When Deg N +1 = DegD

- Deg N2 < DegD2
- A zero atinfinity
- Highest critical frequency is

azero. Case 2: ZLC has zero at s =0

$$Z_{LC}(s) = \frac{N(s)}{D(s)} = \frac{Hs(s^2+\omega_{z1}^2)(s^2+\omega_{z2}^2)\ldots}{(s^2+\omega_{p1}^2)(s^2+\omega_{p2}^2)\ldots} \quad \text{where } 0 < \omega_{p1} < \omega_{z1} < \omega_{p2} < \omega_{z2} < \cdots$$

$$Z_{RC}(s) = [\frac{1}{p} Z_{LC}(p)]_{p^2=s} = \frac{N_2(s)}{D_2(s)} = \frac{H(s+\omega_{z1}^2)(s+\omega_{z2}^2)\ldots}{(s+\omega_{p1}^2)(s+\omega_{p2}^2)\ldots}$$

$$= \frac{H(s+\alpha_1)(s+\alpha_2)\ldots}{(s+\beta_1)(s+\beta_2)\ldots} \quad \text{where } 0 < \beta_1 < \alpha_1 < \beta_2 < \alpha_2 < \cdots$$

- The first critical frequency is a pole at$\beta 1$
- Lowest critical frequency is apole
- Poles and zeros are simple and lie on the negative realaxis
- Poles and zerosinterlace

a) When Deg N = Deg D +1

- Deg N2 = DegD2
- No pole atinfinity
- Highest critical frequency for ZRC is azero

b) When Deg N +1 = DegD

- Deg N2 < DegD2
- A zero atinfinity
- Highest critical frequency is azero.

Note that poles and zeros of ZRC(s) are zeros and poles of YRC(s) respectively.

Summary:

- Poles and zeros are simple and lie on the negative real frequencyaxis.
- Poles and zerosinterlace
- Highest critical frequency is a zero for ZRC and a pole forYRC
- Lowest critical frequency is a pole for ZRC and a zeroYRC.

- Deg N < Deg D for Z and Deg N >

Deg D forY. Ingeneral,

$$Z_{RC}(s) = \frac{H(s+\alpha_1)(s+\alpha_2)\ldots}{s(s+\beta_1)(s+\beta_2)\ldots}$$

$$= \frac{k_0}{s} + \frac{k_1}{s+\beta_1} + \frac{k_2}{s+\beta_2} + \ \ldots + k_\infty \leftarrow \text{if Deg N} = \text{Deg D}$$

$$\frac{dZ_{RC}(\delta)}{d\delta} = \frac{-k_0}{\delta^2} + \frac{-k_1}{(\delta+\delta_1)^2} + \frac{-k_2}{(\delta+\delta_2)^2} + \ \ldots \ \leq 0$$

- $Z(\delta)$ has a negative slope along the real frequency axis. Moreover, *ZRC(s)* has no poles and zeros on the positive real frequency axis as it is a positive realfunction.
- □ $Z(\delta)$ is monotically decreasing from = 0 to∞.
- *Z*(0) >*Z(∞)*
- *Y*(0) <*Y(∞)*.

$$Y_{LC}(s) = \frac{H(s+\omega_{z1}^2)(s+\omega_{z2}^2)\ldots}{s(s+\omega_{p1}^2)(s+\omega_{p2}^2)\ldots} = \frac{k_0}{s} + \frac{k_1 s}{s^2+\omega_{p1}^2} + \frac{k_2 s}{s^2+\omega_{p2}^2} + \ \ldots + Hs$$

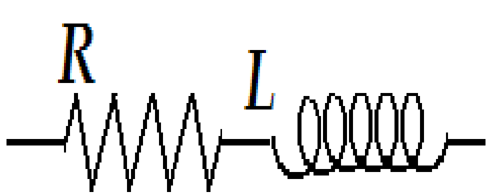
$$Y_{RC}(s) = [pY_{LC}(p)]_{p^2=s} = k_0 + \frac{k_1 s}{s+\omega_{p1}^2} + \frac{k_2 s}{s+\omega_{p2}^2} + \ \ldots + Hs$$

$$(\frac{1}{s})Y_{RC}(s) = \frac{k_0}{s} + \frac{k_1}{s+\omega_{p1}^2} + \frac{k_2}{s+\omega_{p2}^2} + \ \ldots + H$$

$$\left[\frac{1}{s}\right]Y_{RC}(s) = \frac{k_0}{s} + \frac{k_1}{s+\beta_1} + \frac{k_2}{s+\beta_2} + \ \ldots + k_\infty$$

Recall:

The residues at poles of $Z_{Rc}$and Y_{Rc}/s are real and positive.



RL Networks

$Z_{RL}(s)=R_i+L_is=A+Bs$

 Similarly, forLC-network

$Z_{LC}(s) = L_i s + 1/C_i s = \alpha s + 1/\beta s$

Let us s with p

$$Z_{LC}(p) = \alpha p + 1/\beta p$$

$$\Rightarrow \; (p)Z_{LC}(p) = \alpha p^2 + \frac{1}{\beta}$$

$$\Rightarrow \; [(p)Z_{LC}(p)]_{p^2=s} = \frac{1}{\beta} + \alpha s = X + Ys$$

This transformed function has the same form as the RL-network impedance function so that the general RL-network impedance function can be obtained from the general LC-network impedance function.

$$Z_{RL}(s) = [pZ_{LC}(p)]_{p^2=s}$$

Since the impedance and admittance functions of LC networks have similar characteristics, and hence similar forms, we can substitute ZLC with YLC in the above transformation.

$Z_{RL}(s) = [pY_{LC}(p)]_{p^2=s}$ This is similar toY_{RC}

Similarly, the admittance function can be obtained from reciprocal of the impedance function.

$$Y_{RL}(s) = \frac{1}{Z_{RL}(s)}$$

$$= [\frac{1}{p}\frac{1}{Y_{LC}(p)}]_{p^2=s}$$

$Y_{RL}(p) = [\frac{1}{p}Z_{LC}(p)]_{p^2=s}$ This is similar to Z_{RC}

Conclusion:

- Deriving-point RC-impedance and RL-admittance functions have similar forms, and hence similar characteristics.
- Deriving-pointRC-admittanceandRL-impedancefunctionshavesimilarforms,andhence Similar characteristics.

Foster realization of RC networks

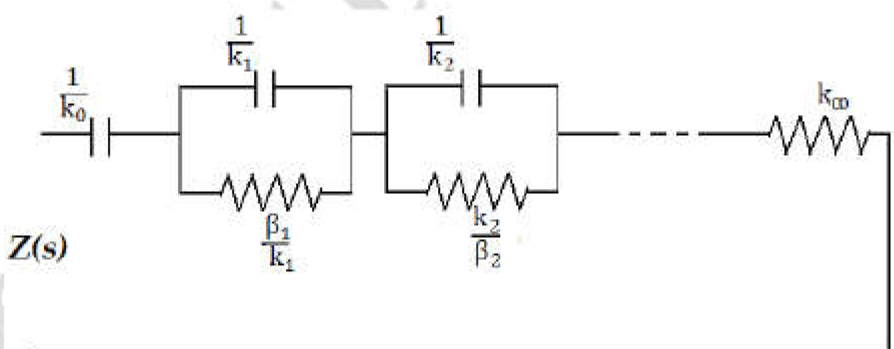
Foster – 1

$$Z_{RC}(s) = \frac{N(s)}{D(s)} \quad Deg\ N(s) \le \ Deg\ D(s)$$

$$Z_{RC}(s) = \frac{H(s+\alpha_1)(s+\alpha_2)\ldots}{s(s+\beta_1)(s+\beta_2)\ldots}$$

$$= \frac{k_0}{s} + \frac{k_1}{s+\beta_1} + \frac{k_2}{s+\beta_2} + \ \ldots + k_\infty \leftarrow \text{if Deg N = Deg D}$$

$$= \frac{1}{\frac{1}{k_0}s} + \frac{1}{\frac{1}{k_1}s + \frac{1}{k_1/\beta_1}} + \frac{1}{\frac{1}{k_2}s + \frac{1}{k_2/\beta_2}} + \ \ldots + k_\infty$$



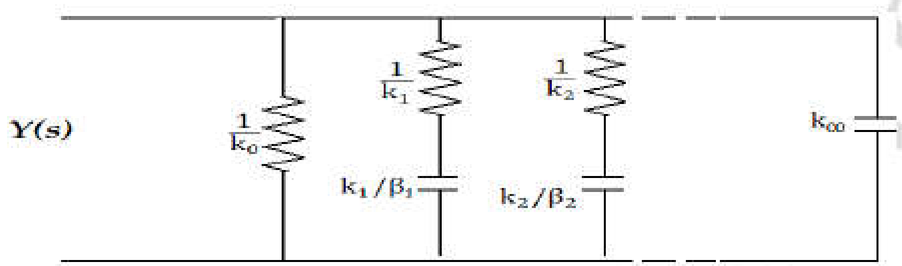
$$Y_{RC}(s) = \frac{H(s+\alpha_1)(s+\alpha_2)\ldots}{(s+\beta_1)(s+\beta_2)\ldots} \quad \text{where } 0 < \alpha_1 < \beta_1 < \alpha_2 < \beta_2 < \cdots$$

$$\left[\frac{1}{s}\right] Y_{RC}(s) = \frac{k_0}{s} + \frac{k_1}{s+\beta_1} + \frac{k_2}{s+\beta_2} + \ \ldots + k_\infty$$

Foster – 2

$$Y_{RC}(s) = k_0 + \frac{k_1 s}{s+\beta_1} + \frac{k_2 s}{s+\beta_2} + \ldots + k_\infty s$$

$$= k_0 + \frac{1}{\frac{1}{k_1} + \frac{1}{k_1/\beta_1 s}} + \frac{1}{\frac{1}{k_2} + \frac{1}{k_2/\beta_2 s}} + \ldots + k_\infty s$$



Foster realization of RL networks

Recall that

- Deriving-point RC-impedance and RL-admittance functions have similar forms, and hence similarcharacteristics.
- Deriving-pointRC-admittanceandRL-impedancefunctionshavesimilarforms,andhence similar characteristics.

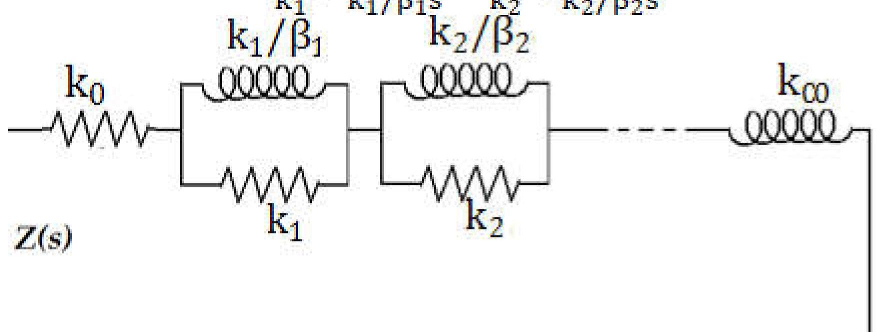
Foster – 1

$$Z_{RL}(s) = \frac{H(s+\alpha_1)(s+\alpha_2)\ldots}{(s+\beta_1)(s+\beta_2)\ldots} \quad \text{where } 0 < \alpha_1 < \beta_1 < \alpha_2 < \beta_2 < \cdots$$

$$\left[\frac{1}{s}\right] Z_{RL}(s) = \frac{k_0}{s} + \frac{k_1}{s+\beta_1} + \frac{k_2}{s+\beta_2} + \ldots + k_\infty$$

$$Z_{RL}(s) = k_0 + \frac{k_1 s}{s+\beta_1} + \frac{k_2 s}{s+\beta_2} + \ldots + k_\infty s$$

$$= k_0 + \frac{1}{\frac{1}{k_1} + \frac{1}{k_1/\beta_1 s}} + \frac{1}{\frac{1}{k_2} + \frac{1}{k_2/\beta_2 s}} + \ldots + k_\infty s$$



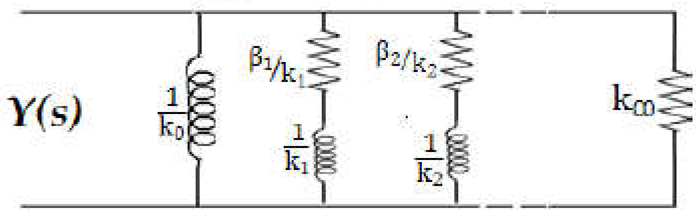
Foster – 2

$$Y_{RL}(s) = \frac{N(s)}{D(s)} \quad Deg\ N(s) \leq \ Deg\ D(s)$$

$$Y_{RL}(s) = \frac{H(s+\alpha_1)(s+\alpha_2)\ldots}{s(s+\beta_1)(s+\beta_2)\ldots}$$

$$= \frac{k_0}{s} + \frac{k_1}{s+\beta_1} + \frac{k_2}{s+\beta_2} + \ \ldots + k_\infty \leftarrow \text{if Deg N} = \text{Deg D}$$

$$Y_{RL}(s) = \frac{1}{\frac{1}{k_0}s} + \frac{1}{\frac{1}{k_1}s + \frac{\beta_1}{k_1}} + \frac{1}{\frac{1}{k_2}s + \frac{\beta_2}{k_2}} + \ \ldots + k_\infty$$



Example 3

Find Foster – 1 and Foster – 2 realization of the following deriving-point function.

$$Z_{RC}(s) = \frac{(s+1)(s+3)(s+5)}{s(s+2)(s+4)(s+6)}$$

Solution:

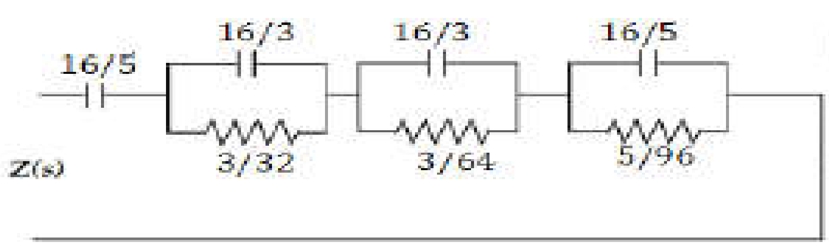
F – 1

$$Deg\ N(s) < \ Deg\ D(s)\ therefore\ k_\infty = 0$$

$$Z_{RC}(s) = \frac{k_0}{s} + \frac{k_1}{s+2} + \frac{k_2}{s+4} + \frac{k_3}{s+6}$$

$$k_0 = \frac{5}{16};\ k_1 = \frac{3}{16} = \ k_2;\ k_3 = 5/16$$

$$Z_{RC}(s) = \frac{1}{16/5s} + \frac{1}{16/3s + \frac{1}{3/32}} + \frac{1}{16/3s + \frac{1}{3/64}} + \frac{1}{16/5s + \frac{1}{5/96}}$$



F – 2

$$Y_{RC}(s) = \frac{s(s+2)(s+4)(s+6)}{(s+1)(s+3)(s+5)}$$

$$\frac{Y_{RC}(s)}{s} = \frac{(s+2)(s+4)(s+6)}{(s+1)(s+3)(s+5)}$$

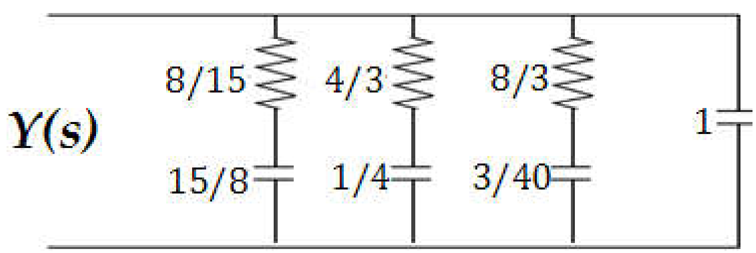
$$= \frac{k_1}{s+1} + \frac{k_2}{s+3} + \frac{k_3}{s+5} + k_\infty$$

$$k_1 = 15/8;\ k_2 = 3/4\ ;\ k_3 = 3/8;\ k_\infty = 1$$

$$\frac{Y_{RC}(s)}{s} = \frac{15/8}{s+1} + \frac{3/4}{s+3} + \frac{3/8}{s+5} + 1$$

$$Y_{RC}(s) = \frac{15/8s}{s+1} + \frac{3/4s}{s+3} + \frac{3/8s}{s+5} + s$$

$$= \frac{1}{8/15 + \frac{1}{15/8s}} + \frac{1}{4/3 + \frac{1}{1/4s}} + \frac{1}{8/3 + \frac{1}{3/40s}} + s$$



Cauer Realization of RC and RL deriving-point impedance function

Cauer realization of a deriving – point function is obtained by Continued Fraction Expansion of the function about the highest or lowest degrees of both the numerator and the denominator polynomials.

For LC deriving-point functions, highest and lowest degrees of the numerator are always deferent (by one) from the highest and lowest degrees of the denominator respectively so that division about the highest or lowest degrees always gives a quotient polynomial of the form *αs* or 1/*αs* that removes a pole at infinity or at zero respectively. Therefore, Cauer realization of LC deriving – point functions is obtained by continued removal ofpoles.

On the other hand, RC and RL deriving – point functions (N(s)/D(s)) can have both numerator and denominator polynomials of the same degree. In this case, division about the highest or lowest degrees gives a constant quotient that removes a constant from the deriving - point function. But to remove a constant from the deriving – point function, the constant must be less than or equal to the minimum of the function. Therefore, it should be known whether from the

impedance or admittance a constant can be
removed. Recall the following properties.

$Z_{RC}(0) > Z_{RC}(\infty)$ ► $Y_{RL}(0) > Y_{RL}(\infty)$ *since* Z_{RC} *and* Y_{RL} *have similar*

Z_{RC} *and* Y_{RL} have minimum values at infinity (∞) so that a constant can be removed duringCauer-1 realization (continued fraction expansion about infinity).

Y_{RC} *and* Z_{RL} have minimum values at zero (0) so that a constant can be removed during Cauer-2 realization (continued fraction expansion about zero).

Cauer – 1 realization

For RC networks use the impedance function, *ZRC*, and for RL network use the admittance function, Y_{RL}.

$$Z_{RC}(s) = \frac{N(s)}{D(s)} = Y_{RL}(s)$$

Deg N(s) < Deg D(s) ► No pole at infinity <u>Case 1: Deg N = Deg D</u>

Since these functions have minimum values at infinity, a constant can be removed by division about infinity (highest degrees). This leaves a remainder function whose numerator has a degree less that the denominator, and hence has zero at infinity. This can be removed as a pole from the reciprocal. Now the second remainder function will have both numerators and denominators of the same degree. Now a constant (at infinity) cannot be removed from these functions! But a constant (at infinity) can be removed from the reciprocal since it has a minimum at infinity.

Let us assume the function is Z_{RC} (the same conclusion can be reached for Y_{RL}).

A constant at infinity can be removed from ZRC not YRC. Now after a constant is removed from Z, the remainder *Z2* will have a numerator with its degree less than degree of the denominator so that Z2 will have zero at infinity. This implies that Y2 = 1/Z2 has a pole at infinity. After a pole is removed from Y2, the remainder Y3 will again have numerator and denominator with the same degree. Since a constant (at infinity) cannot be removed from YRC (and ZRL) we take the reciprocal of Y3 (Z3) and remove a constant. The expansion continues by applying the same process.

Cauer – 1 form of ZRC and YRL will be as shown below.

$$Z_{RC}(s) = \beta_1 + \cfrac{1}{\beta_2 s + \cfrac{1}{\beta_3 + \cfrac{1}{\beta_4 s + \cdots}}} = Y_{RL}(s)$$

<u>Case 2: Deg N < Deg D zero at infinity</u>

In this case there is no constant removal at infinity (the function becomes zero at infinity) so that the expansion starts from the reciprocal function. $\beta_1 = 0$

$$Z_{RC}(s) = \cfrac{1}{\beta_2 s + \cfrac{1}{\beta_3 + \cfrac{1}{\beta_4 s + \cdots}}} = Y_{RL}(s)$$

Cauer – 2 realizations

For RC networks use the admittance function, *YRC*, and for RL network use the impedance function,

$$Y_{RC}(s) = \frac{N(s)}{D(s)} = Z_{RL}(s) \qquad \text{Deg N} \geq \text{Deg D}$$

ZRL.

These functions have minimum values at zero so that a constant about the origin (at zero) can be removed by dividing about the lowest degrees.

<u>Case 1: zero at s = 0</u>

No constant removal at zero since the function becomes zero at s = 0. However, the reciprocal will have pole at zero. This pole is removed first and the remainder will have no pole at zero anymore. Now a constant at zero is removed from the reciprocal. Removal of a constant at zero leaves another remainder which has zero at s = 0 (since the constant is already removed). Therefore, a pole at zero is removed from reciprocal of this function. The expansion continues by following the same procedure.

$$Y_{RC}(s) = \cfrac{1}{\cfrac{1}{\alpha_1 s} \cfrac{1}{\cfrac{1}{\alpha_2} + \cfrac{1}{\cfrac{1}{\alpha_3 s} + \cfrac{1}{\cfrac{1}{\alpha_4} + \cdots}}}} = Z_{RL}(s)$$

Finally, Caurer as shown below. <u>Case 2: no zero at s = 0</u>

In this case a constant at zero is removed first.

$$Y_{RC}(s) = a + \cfrac{1}{\cfrac{1}{a_1 s} + \cfrac{1}{\cfrac{1}{a_2} + \cfrac{1}{\cfrac{1}{a_3 s} + \cfrac{1}{\cfrac{1}{a_4} + \cdots}}}} = Z_{RL}(s)$$

Example 5

Realize the following RL deriving-point function using Cauer – 1 and Cauer – 2

$$Z_{RL}(s) = \frac{s^2 + 5s + 4}{3s + 4}$$

Solution:

$Z_{RL}(0) < Z_{RL}(\infty)$

$Y_{RL}(\infty) < Y_{RL}(0)$

For Cauer – 1 (Continued Fraction Expansion about infinity), use *YRL* For Cauer – 2 (Continued Fraction Expansion about the origin) use *ZRL* Cauer – 1

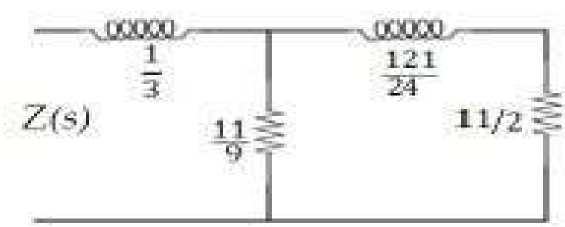
$$Y_{RL}(s) = \frac{3s + 4}{s^2 + 5s + 4}$$

Deg N < Deg D

- No pole or constant removal atinfinity
- Start from the reciprocal function(*ZRL*)

$$Z_{RL}(s) = \frac{s^2 + 5s + 4}{3s + 4}$$

$$= \frac{1}{3}s + \cfrac{1}{\cfrac{9}{11} + \cfrac{1}{\cfrac{121}{24}s + \cfrac{1}{2/11}}}$$



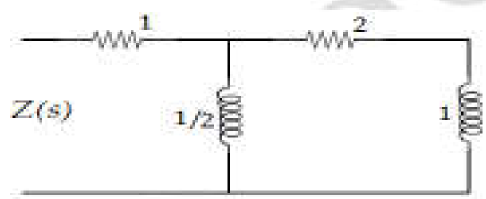
Cauer – 2

$$Z_{RL}(s) = \frac{s^2 + 5s + 4}{3s + 4}$$

- No zero at zero

$$Z_{RL}(s) = \frac{1}{3}s + \cfrac{1}{\frac{9}{11} + \cfrac{1}{\frac{121}{4}s + \cfrac{1}{2/11}}}$$

$$= 1 + \cfrac{1}{\frac{2}{s} + \cfrac{1}{2 + \cfrac{1}{1/s}}}$$



A constant can be removed at zero.

Code:80203

MALLA REDDY ENGINEERING COLLEGE (AUTONOMOUS)

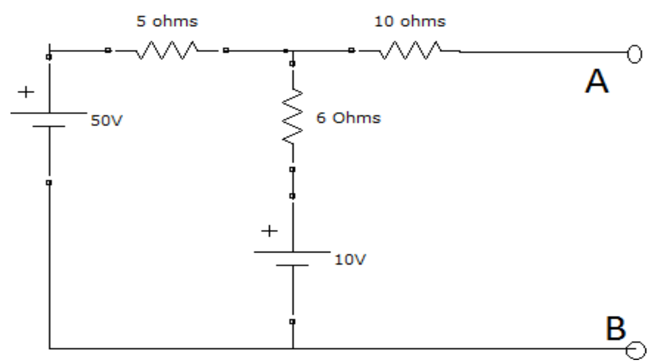
II B.Tech I Semester I Mid Question Bank 2019-20

Subject:ECAS **Branch: EEE**

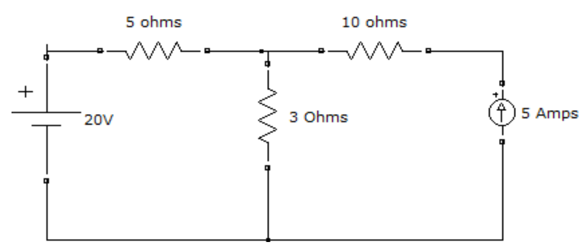
Name of the Faculty: Y Sudha

MODULE-I

1. Find Thevenins Equivalent circuit for the following circuit



2. State and Explain Nortons theorem and Tellegens theorem?
3. Proof R_{th}=$R_L$for Maximum power transfer theorem
4. Solve this network and find current passing 3Ω using superposition theorem.



5. Two coils of number of turns N_1=1000,N_2=400 respectively are placed near each other.They are magnetically coupled in such away that 75% of flux produced by one of 1000 turns links other.A current of 6amp produces a flux of 0.8mwb in N_1 and same amount of current produces a flux of 0.5mwb in the coil of N_2 turns.Determine L_1,L_2,M & K for coils?
6. Define composite Magnetic circuit? Explain about Parallel magnetic circuit with neat diagram?s

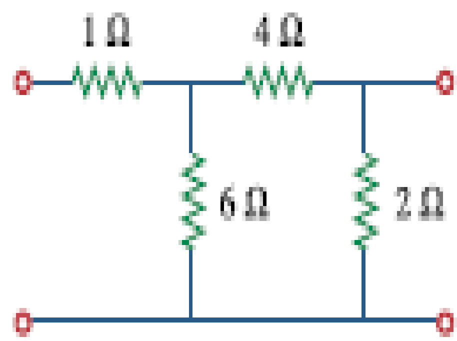
MODULE-II

1. Explain about parallel resonance and derive Bandwidth and quality Factor for it?
2. Explain about the three phase systems and advantages of three phase system?
3. Explain about representation of a three phase system for line and phase voltages and currents in a star connection for a balanced system?
4. A rms line voltage in a three phase start circuit is given by 213V(P-N).Write the instantaneous voltage expression.If the currents in each phase lag the corresponding phase voltages by 30° ,What are the expressions of intantaneous currents?

5. Explain how three phase power is measured using Two Wattmeter method with corresponding equations and diagram.
6. A Three phase 440V load has a power factor of 0.4.The two wattmeters are connected to measure the power.If theinput power be 10KW find the reading of each instrument?

MODULE-III

1. Explain briefly about Z-parameters with relevant equations and diagrams.
2. Explain and derive Y-parameters with relevant equations?
3. Determine Z-parameters for the following circuit



MALLA REDDY ENGINEERING COLLEGE (AUTONOMOUS)
B.Tech– II YEAR I Sem (MR 18)
II Mid Examination Subjective Question Bank

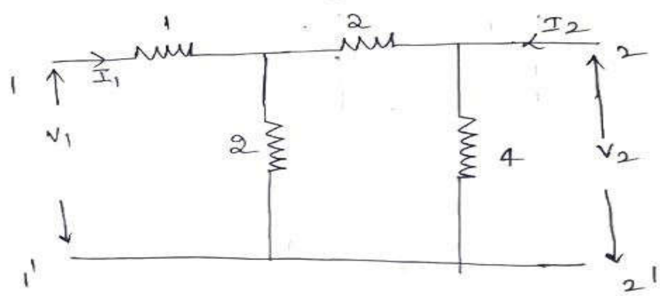
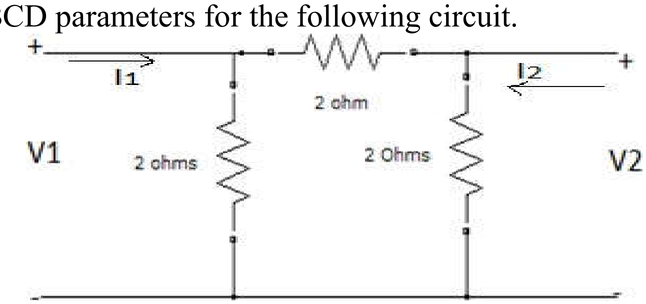
Subject:ECAS **Branch:EEE**

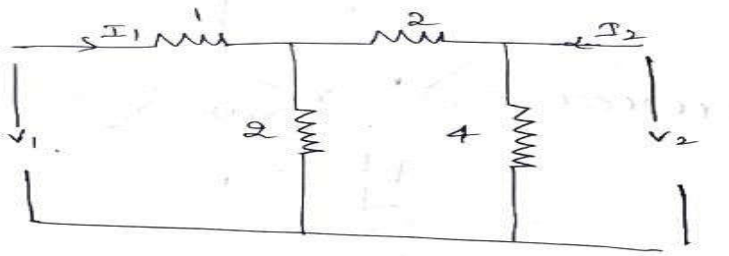
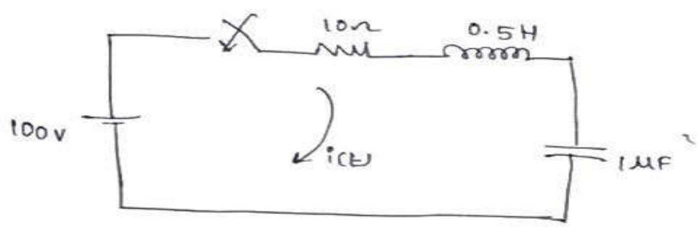
Name of the faculty: Y SUDHA

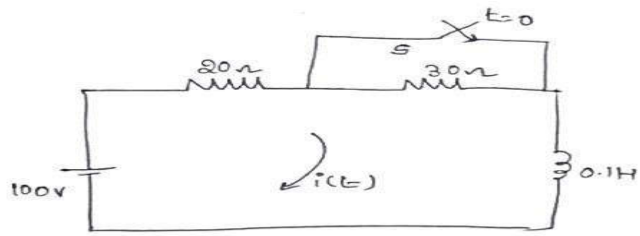
<u>Instructions:</u>

1. All the questions carry equal marks

2. Solve all the questions

<table>
<tr><td colspan="4"><u>Module III</u></td></tr>
<tr><th>Q.No.</th><th>Question</th><th>Bloom's Taxonomy Level</th><th>CO</th></tr>
<tr><td>1.</td><td>Solve the h-parameters of the network shown in figure.

</td><td>Applying</td><td>3</td></tr>
<tr><td colspan="4">OR</td></tr>
<tr><td>2.</td><td>lve ABCD parameters for the following circuit.

</td><td>Applying</td><td>3</td></tr>
<tr><td>3.</td><td>List the Relationship between Z parameter interms of Y,ABCD & H parameters.</td><td>Analyzing</td><td>3</td></tr>
<tr><td colspan="4">OR</td></tr>
</table>

4.	Examine the Y-Parameter foe the network shown in figure. 	Analyzing	3
<u>Module IV</u>			
1.	Explain the DC response of an RL series circuit with intital conditions using laplace transforms.	Understanding	4
OR			
2.	A series RLC circuit shown below comprising R=10Ω,L=0.5H,C=1μF is excited by a constant voltage source of 100V. Determine the expression for the current.Assume the expression for the current.Assume that the circuit is relaxed initially. 	Evaluating	4
3.	Explain the DC response of an RLC series circuit with intital conditions using laplace transforms.	Understanding	4
OR			
4.	Explain the Sinusoidal response of an RL series circuit with intital conditions using laplace transforms.	Understanding	4
5.	Explain the Sinusoidal response of an RC series circuit with intital conditions using laplace transforms.	Understanding	4
OR			
6.	Explain the step response of an RL and RC parallel sinusoidal circuits.	Understanding	4
7.	Explain the DC response of an RC series circuit with intital conditions using laplace transforms.	Understanding	4
OR			
8.	For the circuit shown in figure . Examine the complete	Analyzing	4

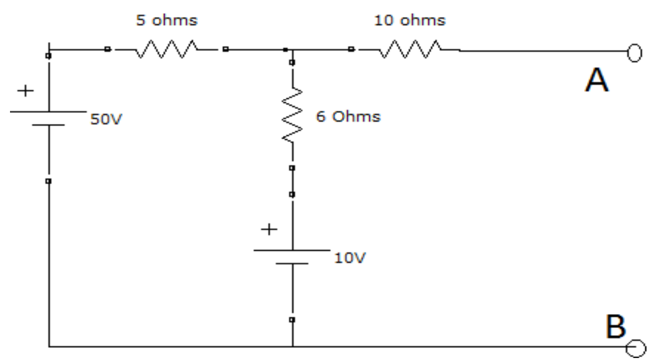
	expression for the circuit when the switch is closed at t=0 		
Module V			
1.	plain about the synthesis of R-L circuit by cauer method with an example.	Understanding	5
	OR		
2.	Illustrate the polynomial $P(s) = s^4+3s^2+2$ is Hurwitz or not.	Understanding	5
3.	e driving point impedance of an LC network is given by Z(s)= $2s^5+12s^3+16s$. Determine cauer first form of network. $+4s^2+3$	Evaluating	5
	OR		
4.	lve whether the polynomial $P(s) = s^4+s^3+3s^2+2s+12$ is Hurwitz.	Applying	5
5.	Simplify the first Foster form of the driving point function of $Z(S)=2(S+2)(S+5)/(S+4)(S+6)$	Analyzing	5
	OR		
6.	Explain about Synthesis of Reactive one-Ports by Fosters method.	Understanding	5
7.	Explain about Synthesis of Reactive one-Ports by Cauer method.	Understanding	5
	OR		
8.	Simplify the tow Cauer realisations of driving point function given by $Z(S)=(10S^4+12S^2+1)/2S^3+2S$.	Analyzing	5

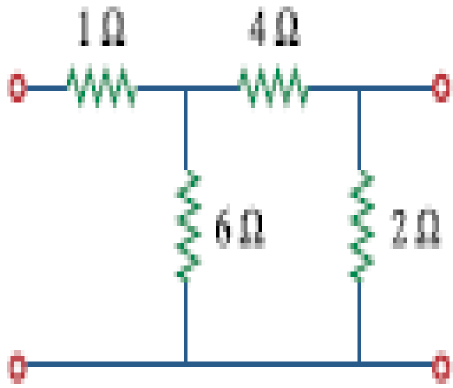
Signature of the Faculty **Signature of the HoD**

MALLA REDDY ENGINEERING COLLEGE (AUTONOMOUS)

B.Tech– II YEAR I Regular Examinations (MR 18)
Model Question Bank

Name of the Subject:ECAS **Branch:EEE**
Name of the faculty: Y SUDHA

Q.No.	Question	Bloom's Taxonomy Level	CO
1.	illustrate Thevenins Equivalent circuit for the following circuit 	Understanding	1
OR			
2.	Two coils of number of turns N_1=1000,N_2=400 respectively are placed near each other.They are magnetically coupled in such away that 75% of flux produced by one of 1000 turns links other.A current 0f 6Amp produces the flux of 0.8 mwb in N_1 and same amount of current produces of flux of 0.5mwb in the coil of N_2 turns.Determine L_1,L_2,M and K for coils?	Understanding	1
3.	plain about parallel resonance and derive Bandwidth and quality Factor for it?	Understanding	2
OR			
4.	plain about representation of a three phase system for line and phase voltages and currents in a star connection for a balanced system?	Understanding	2
5.	a)Choose Z-parameters with relevant equations and diagrams Solve Z-parameters for the following circuit	Applying	3

			
	OR		
6.	Solve the series and parallel Interconnection of two port network	Applying	3
7.	Derive the DC response of an RC series circuit with initial conditions by applying laplace transform?	Analyzing	4
	OR		
8.	A 50KZ 400V(Peak value) sinusoidal voltage is applied at t=0 to a series RL circuit having resistance 5ohms and inductance 0.2H.Obtain the expression for current at any instant "t"Examine the value of the transient current 0.01sec after switching on?	Analyzing	4
9	e Driving point impedance of an LC network is given by$Z(S)=(S^4+4S^2+3)/(S^3+2S)$.Determine the Second Cauer Form of the network?	Evaluating	5
	OR		
10	plain about Synthesis of Reactive one-Ports by Fosters method?	Evaluating	5

Signature of the Faculty **Signature of the HoD**